Supplement (Trigonometric, Exponential, and Logarithm functions)

- **Trigonometric Functions**

  You have one formula that is useful in many cases.

$$\sin^2 x + \cos^2 x = 1$$

  Another formulas helpful if you know are

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \tag{1}$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y \tag{2}$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y \tag{3}$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y \tag{4}$$

  In fact, (2) can be induced from (1) *and* (4) can be derived from (3). The formula below is the result of dividing (1) by (3),

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}.$$

  If we set $y = x$ in (1), (3), we get some useful formula,

$$\sin 2x = 2 \sin x \cos x \tag{5}$$
$$\cos 2x = \cos^2 x - \sin^2 x \tag{6}$$

  With the fact that $\sin^2 x + \cos^2 x = 1$ holds, (6) have alternative versions (7) and (8),

$$\cos 2x = 2\cos^2 x - 1 \tag{7}$$
$$= 1 - 2\sin^2 x \tag{8}$$

- **the Exponential Function and the Logarithm Function**

  The exponential function satisfies

$$e^a \times e^b = e^{a+b}. \tag{9}$$

  Along with the exponential function, there is the inverse function of the exponential functions called the logarithm function and denoted by $\ln x$. It is the number satisfying

$$e^{\ln x} = x.$$

  The definition of a^n is $e^{n \times \ln a}$ for any positive real number a. In addition, using (9), you get

$$\ln a + \ln b = \ln(a \times b), \quad -\ln a = \ln\left(\frac{1}{a}\right)$$

- **Derivatives**

$$\frac{d}{dx}(\sin cx) = c\cos cx, \quad \frac{d}{dx}(\cos cx) = -c\sin cx, \quad \frac{d}{dx}(\tan cx) = -\frac{c}{\cos^2 cx}$$

$$\frac{d}{dx}(e^{cx}) = ce^{cx}, \quad \frac{d}{dx}(\log cx) = \frac{1}{x}$$