1. Use the energy integral lemma to show that every solution ot the Duffing equation

$$y'' + y + y^3 = 0$$

is bounded; that is, $|y(t)| \leq M$ for some M.

2. (Vandermonde matrix) Prove that the determinant of a Vandermonde matix

$$\begin{pmatrix} 1 & \alpha_1 & \alpha_1^2 & \dots & \alpha_1^{n-1} \\ 1 & \alpha_2 & \alpha_2^2 & \dots & \alpha_2^{n-1} \\ 1 & \alpha_3 & \alpha_3^2 & \dots & \alpha_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_n & \alpha_n^2 & \dots & \alpha_n^{n-1} \end{pmatrix}$$

is

$$\prod_{1 \leq i < j \leq n} (\alpha_j - \alpha_i).$$