1 The matrix equation $A\mathbf{x} = \mathbf{b}$

1.1 Multiplying a vector $\mathbf{x}$ to a matrix A

One matrix equation represents one linear system!

1.2 Theorem 4

Let A be an $m \times n$ matrix. Then the following statements are logically equivalent.

a. For each $\mathbf{b}$ in $\mathbb{R}^m$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution.
b. Each $\mathbf{b}$ in $\mathbb{R}^m$ is a linear combination of the columns of A.
c. The columns of A span $\mathbb{R}^m$.
d. A has a pivot position in every row.

1.3 Solutions of a linear system (revisit)

1.3.1 Homogeneous linear systems

A system of linear equations is said to be ***homogeneous*** if it can be written in the form $A\mathbf{x} = \mathbf{0}$. This equation always has at least one solution, namely, $\mathbf{x} = \mathbf{0}$ which is usually called the **trivial solution**. So, we are interested in finding all **nontrivial solutions**.

1.3.2 Nonhomogeneous linear systems

Here is **Theorem 6**

> If the equation $A\mathbf{x} = \mathbf{b}$ is *consistent* for some given $\mathbf{b}$, and let $\mathbf{p}$ be a solution (any possible solution). Then the solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is any solution of the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

2 Linear Independence

THIS CONCEPT IS VERY IMPORTANT!

An indexed set of vectors $\{v_1, \cdots, v_n\}$ in $\mathbb{R}^m$ is said to be ***linearly independent*** if the vector equation

$$x_1 v_1 + x_2 v_2 + \cdots + x_n v_n = 0$$

has only the *trivial solution*. It is ***linearly dependent*** if there exist x_1, x_2, $\cdots$, x_n, not all zero, satisfying the above vector equation.

In other words, $\{v_1, \cdots, v_n\}$ is *linearly dependent* if one of the vectors is the linear combination of other vectors. There could be one tautological definition of that using the concept of *Span*.

There is one particular case ; one of the vectors is the zero vector. In this case, the set of the vectors is *linearly dependent* always.