Quiz 8

APRIL 16, 2019

Name (Last, First)

DongGyu Lim

1. (5pts) Solve the given initial value problem:

$$y'' + 2y' + 5y = 0; \quad y(0) = 2, \quad y'(0) = -4.$$

Solution. The auxiliary equation is

$$r^2 + 2r + 5 = 0$$

and the roots will be $-1 + 2i$ and $-1 - 2i$. So, we know that any solution would be of the form

$$y(t) = c_1 e^{-t} \cos 2t + c_2 e^{-t} \sin 2t.$$

As the initial values are given, we can plug in $t = 0$ and do the same thing for $y'(t)$. Hence, we have

$$2 = y(0) = c_1 \qquad \text{and} \qquad -4 = y'(0) = -c_1 + 2c_2.$$

Hence, $c_1 = 2$ and $c_2 = -1$. The unique solution $y(t)$ is $2e^{-t} \cos 2t - e^{-t} \sin 2t$.

2. (5pts) Find the general solution to the differential equation:

$$\frac{d^2 y}{dx^2} + 4y = 16x \sin 2x.$$

Solution. The auxiliary equation is

$$r^2 + 4r = 0$$

and the roots are $\pm 2i$. Hence, according to the method of undetermined coefficients, we need to try

$$y_p(x) = x(ax + b) \cos 2x + x(cx + d) \sin 2x$$

and find for a, b, c, d. We will just compute $y_p'(x)$ and $y_p''(x)$:

$$y_p'(x) = (2ax + b + 2cx^2 + 2dx) \cos 2x + (2cx + d - 2ax^2 - 2bx) \sin 2x$$

and

$$y_p''(x) = (4cx + 2d + 2a + 4cx + 2d - 4ax^2 - 4bx) \cos 2x + (2c - 4ax - 2b - 4ax - 2b - 4cx^2 - 4dx) \sin 2x.$$

Now,

$$y_p''(x) + 4y_p(x) = (8cx + 2a + 4d) \cos 2x + (-8ax - 4b + 2c) \sin 2x.$$

We want the right hand side to be $16x \sin 2x$. So, $8cx + 2a + 4d = 0$ and $-8ax - 4b + 2c = 16x$. If you think about x-terms and constant terms separately, you get $c = 0$, $a = -2$, $2a + 4d = 0$, $-4b + 2c = 0$ so that $a = -2$, $b = 0$, $c = 0$, $d = 1$. The general solution now is

$$y_p(x) + \text{homogeneous case solutions} = -2x^2 \cos 2x + x \sin 2x + c_1 \cos 2x + c_2 \sin 2x.$$