Quiz 3

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Name (Last, First)

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1. (8pts) Let A be the matrix given below:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & -3 & 1 \\ 3 & 7 & 13 \end{bmatrix}.$$

a. Find a basis of the column space of A.

b. What is the rank of A?

c. Compute the dimension of the null space of A.

Solution. As pivot columns form a basis of the column space of A, we first need to find pivot columns.

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & -3 & 1 \\ 3 & 7 & 13 \end{bmatrix} \xrightarrow[R_2 \to R_2/(-7)]{\substack{R_3 \to R_3 - 3R_1 \\ R_2 \to R_2 - 2R_1}} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

So, the first and second columns of A are pivot columns.
a.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix} \right\} \text{ is a basis of Col}A.$$

b. rk$A = \dim$Col$A = \#$ of vectors in a basis of Col$A = 2$.
c. The rank theorem tells us that we have

$$\text{rk}A + \dim \text{Nul}A = n.$$

In this problem, $n = 3$ and from b. we know rk$A = 2$. So, the dimension of the null space of A is 1.

2. (2pts) Let $M_{2\times 2}$ be the vector space of all 2×2 matrices. Find the coordinate vector of $v = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ relative to the basis $\mathcal{B}$ of $M_{2\times 2}$: (blanks are 0's)

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & \\ & \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ & \end{bmatrix}, \begin{bmatrix} 1 & \\ 1 & \end{bmatrix}, \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \right\}.$$

Solution. The definition of the coordinate vector relative to $\mathcal{B}$ is the vector written in the vertical way with entries the same as the coefficients. So, we need to find x_1, x_2, x_3, and x_4 satisfying

$$\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} = x_1 \begin{bmatrix} 1 & \\ & \end{bmatrix} + x_2 \begin{bmatrix} 1 & 1 \\ & \end{bmatrix} + x_3 \begin{bmatrix} 1 & \\ 1 & \end{bmatrix} + x_4 \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}.$$

You can compare each entries now. (2,2)-entry gives you $x_4 = 1$. (2,1)-entry gives you $x_3 = -3$. (1,2)-entry gives you $x_2 = 5$. Now, (1,1)-entry of the left hand side is 2 and it is $x_1 + x_2 + x_3 + x_4 = x_1 + 5 + (-3) + 1$ on the right hand side so that $x_1 = -1$. Now, the answer is

$$[v]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 5 \\ -3 \\ 1 \end{bmatrix}.$$