Quiz 2

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Name (Last, First)

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1. (8pts) Let A be the matrix given below:

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & -4 & 1 \\ -1 & 3 & -1 \end{bmatrix}.$$

a. Find the determinant of A.

b. Find the solution for $A\mathbf{x}_1 = \mathbf{e}_1$ and $A\mathbf{x}_2 = \mathbf{e}_2$ and $A\mathbf{x}_3 = \mathbf{e}_3$.

c. Find the inverse matrix of A.

Solution. One can solve all these problems by doing the row reduction process only using replacement and interchange, but not scaling. In the final step, we will scale if needed.

$$\begin{bmatrix} 4 & 2 & 3 & 1 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 \\ -1 & 3 & -1 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[R_1 \to R_1 - 3R_2 + 2R_3]{\substack{R_3 \to R_3 + R_2 \\ R_2 \to R_2 - 4R_3}} \begin{bmatrix} 1 & 0 & 0 & 1 & 11 & 14 \\ 1 & 0 & 1 & 0 & -3 & -4 \\ 0 & -1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow{\substack{R_2 \to R_2 - R_1 \\ R_2 \leftrightarrow R_3}} \begin{bmatrix} 1 & 0 & 0 & 1 & 11 & 14 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -14 & -18 \end{bmatrix} \xrightarrow{R_2 \to (-1) \times R_2} \begin{bmatrix} 1 & 0 & 0 & 1 & 11 & 14 \\ 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 & -14 & -18 \end{bmatrix}$$

a. You need to count the number of interchange you have done, which is 1, and how much you have scaled, which is (-1) to get the identity matrix I_3 on the left half of the 3×6 matrix. So, the determinant of A is $(-1)^1 \times \frac{1}{(-1)} \times \det I_3 = 1$. Or, of course, you can compute the determinant at some point after only doing the replacement.
b. You can look at the "appropriate" 3×4 matrix which actually is the augmented matrix for each equation: 1,2,3,4th columns for $A\mathbf{x}_1 = \mathbf{e}_1$. 1,2,3,5th columns for $A\mathbf{x}_2 = \mathbf{e}_2$. 1,2,3,6th columns for $A\mathbf{x}_3 = \mathbf{e}_3$. In other words, $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ are the 4,5,6th columns of the reduced row echelon form.
c. You can just copy the right half of the final 3×6 matrix. Done!

2. (2pts) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Compute AA^T and A^TA and check if they are invertible or not.

Solution.

$$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad A^TA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

So, AA^T is invertible because it is the identity matrix I_2. However, A^TA is not as the $\det A^TA$ is 0 (cofactor expansion using the 3rd column).