Quiz 1

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Name (Last, First)

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1. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 3 \\ -8 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix}$. Does the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ contain the following vector? Why or why not?

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Solution. In general, we can check if $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is in the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Another way to say this is[1] if the augmented matrix

$$\begin{bmatrix} 1 & 0 & 4 & a \\ 0 & 3 & -1 & b \\ 1 & -8 & 7 & c \end{bmatrix}$$

is from a consistent linear system. Applying some row operations, we get

$$\begin{bmatrix} 1 & 0 & 0 & f \\ 0 & 1 & 0 & g \\ 0 & 0 & 1 & h \end{bmatrix}$$

where f, g, h are some functions of a, b, c. In fact, $h(a, b, c) = -3a + 8b + 3c$. However, no matter what f, g, h are, this implies that the system is consistent. Hence, $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ spans $\mathbb{R}^3$ so that it contains the vector above.

2. The following statement is false. **Construct a specific example** to show that the statement is not always true. (Caution: *Please read carefully. This is different from your HW problem.*)

If $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are in $\mathbb{R}^4$ and $\mathbf{v}_3$ is not a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.

Solution. First of all, read the question carefully. One can see that usually the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ would be linearly independent. Now, we need to find a counterexample to prove that the statement is false. Now try to make 3 linearly independent vectors in $\mathbb{R}^4$ as following:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

It is not hard to check if they are linearly indepdendent[2] and $\mathbf{v}_3$ is not a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

[1] Using the matrix equation format $A\mathbf{x} = \mathbf{b}$.
[2] Just build up the equation condition for linear dependence.