Quiz 7

OCTOBER 24, 2019

Name (Last, First)

Answer Key

1. (6pts) Find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that the given matrix

$$A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix}$$

has the form $A = PCP^{-1}$.

$$\begin{bmatrix} 1-\lambda & 5 \\ -2 & 3-\lambda \end{bmatrix}$$

$$(1-\lambda)(3-\lambda)+10=0$$

$$3-4\lambda+\lambda^2+10=0$$

$$\lambda^2-4\lambda+13=0$$

$$\lambda = \frac{4 \pm \sqrt{16-52}}{2} = \frac{4\pm 6i}{2} = 2\pm 3i \quad a=2, b=3$$

for $\lambda = 2+3i$,

$$A_1 = \begin{bmatrix} 1-(2+3i) & 5 \\ -2 & 3-(2+3i) \end{bmatrix} = \begin{bmatrix} -1-3i & 5 \\ -2 & 1-3i \end{bmatrix}$$

$$(-1-3i)x_1 + 5x_2 = 0$$

$$-2x_1 + (1-3i)x_2 = 0$$

$$x_1 = \frac{-5}{-1-3i}x_2 = \frac{5}{1+3i}x_2$$

rank(A_1) must be 1, so Span 1 vector in nullspace.

$$\mathrm{Nul}(A_1) = \mathrm{Span}\left\{\begin{bmatrix} 1-3i \\ 2 \end{bmatrix}\right\}, \quad \alpha = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \beta = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

$$\mathrm{Nul}(A_1) = \mathrm{Span}\left\{\begin{bmatrix} \frac{5}{1+3i} \\ 1 \end{bmatrix}\right\} = \mathrm{Span}\left\{\begin{bmatrix} 5 \\ 1+3i \end{bmatrix}\right\}$$

could also use this as α, β where

$$P = \begin{bmatrix} 5 & 0 \\ 1 & -3 \end{bmatrix}$$

w/ same C.

$$C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 3 & 2 \end{bmatrix}$$

$$P = \begin{bmatrix} \alpha & -\beta \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$$

2. (4pts) Find the angle between u and v for

$$u = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}.$$

$$\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \sqrt{1+1}\,\sqrt{1+4+4}\,\cos\theta$$

$$1\cdot 1 + 0\cdot 2 + 1\cdot 2 = \sqrt{2}\cdot 3\cos\theta$$

$$3 = 3\sqrt{2}\cos\theta$$

$$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}$$

← $\frac{\pi}{4}, -\frac{\pi}{4}$ are same relative to $\vec{u}, \vec{v}$ vectors.