Quiz 5

SEPTEMBER 26, 2019

Name (Last, First)

Answer Key

WARNING. Please be as explicit as possible. For example, you should NOT just say $\{A\mathbf{x}\}$ for the definition of Col$A$. Instead, you should say

$\{A\mathbf{x}$ for all $\mathbf{x} \in \mathbb{R}^n\}$ OR 'the set of all linear combinations of the columns of A'.

Don't forget to use the expressions like 'for all' or 'for some' appropriately.

1. (6pts) Write down the **definitions** of the following concepts and give some **examples**.

a. The set of vectors $\{v_1, \cdots, v_r\} \subset \mathbb{R}^n$ is linearly independent.

The set of vectors that are not linear combinations of each other.

ex. $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}$

b. The set of vectors $\{v_1, \cdots, v_r\}$ is a basis for a vector space V.

The set of vectors that form a linearly independent set in V that spans V.

ex. $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$ is a basis for the space R^3

c. The linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is onto.

The mapping $T: R^n \to R^m$ is onto if each $\vec{b}$ in R^m is the image of at least one x in R^n. For every y in Rm, there is at least one x in R^n such that $f(x)=y$.

ex. $T = \begin{bmatrix}1&0&0\\0&1&2\end{bmatrix}$

2. (4pts) Mark all sentences that are "grammatically" incorrect.

a. The matrix $\begin{bmatrix}a&b\\c&d\\e&f\end{bmatrix}$ is linearly independent.

X - only a set of vectors can be linearly independent/dependent, not a matrix.

b. The columns of a matrix $\begin{bmatrix}a&b\\c&d\end{bmatrix}$ form a basis for $\mathbb{R}^2$.

"the columns of a matrix A are linearly independent" works instead.

c. For a matrix A, Nul$A$ is a vector space.

d. The matrix $\begin{bmatrix}a&b&c\\d&e&f\end{bmatrix}$ has dimension 3.

X - dimension (and basis) are only applicable to a vector space. Instead, something like "The vector space of all 3x3 matrices has dimension 3" works.

e. The set of vectors $\left\{\begin{bmatrix}2\\0\end{bmatrix}, \begin{bmatrix}0\\-1\end{bmatrix}\right\}$ is one-to-one.

X - only a transformation can be one-to-one, not a set of vectors.

f. The matrix transformation defined by $\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}$ is onto.

"The matrix transformation defined by A is one-to-one"