Quiz 3

SEPTEMBER 12, 2019

Name (Last, First)

Answer Key

In the following problems, A is the matrix given by

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 4 & 5 \\ -2 & 7 & 8 \end{bmatrix}.$$

1. (7pts) Find the null space of A. (Find Nul A.) Are the columns of A linearly independent or dependent?

The nullspace of a matrix A is the set of all solutions to the homogeneous equation $A\vec{x}=\vec{0}$. Because Nul(A) is defined by a condition that must be checked, we find Nul(A) by augmenting the zero vector onto matrix A.

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 4 & 5 \\ -2 & 7 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 4 & 5 & 0 \\ -2 & 7 & 8 & 0 \end{array}\right] \xrightarrow[R_3 \to 2R_1 + R_3]{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & 5 & 10 & 0 \end{bmatrix} \xrightarrow[R_3 \to 5R_2 - 3R_3]{R_2 \to \frac{1}{2}R_2} \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 \to R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \begin{aligned} x_1 + 3t &= 0 \\ x_2 + 2t &= 0 \\ x_3 &= t \end{aligned} \to \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} t \text{ for all } t \in \mathbb{R}$$

non-pivot column, there must be a free variable

$\therefore$ $\text{Nul}(A) = \text{Span}\left\{\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}\right\}$ and b/c nullspace is not just the trivial solution, the columns of A are linearly dependent.

2. (3pts) For $\mathbf{b} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$, it is known that $\mathbf{x} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ is a particular solution of the matrix equation $A\mathbf{x} = \mathbf{b}$. Find the general solution of the matrix equation $A\mathbf{x} = \mathbf{b}$. (Hint. What is the relation between the general solutions of homogeneous and nonhomogeneous equations?)

Theorem 6 (book section 1.5): Solution set of $A\vec{x}=\vec{b}$ is the set of all vectors of the form $\vec{w} = \vec{p} + \vec{v}_h$ where $\vec{v}_h$ is any solution of the homogeneous equation $A\vec{x}=\vec{0}$.

Since we know solution set of $A\vec{x}=\vec{0}$ from problem 1, we have $\vec{v}_h$.

$$A\vec{x} = b\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} + \underbrace{\text{Span}\{\ \}x : A\vec{x} = \vec{0}}_{\text{Nul}(A)}$$

$$= \left\{ \underbrace{\begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}}_{\vec{p}} + \underbrace{\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}}_{\vec{v}_h} t \text{ for } t \in \mathbb{R} \right\}$$