Quiz 2

SEPTEMBER 5, 2019

Name (Last, First)

Answer Key

1. (7pts) Determine if $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$.

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}.$$

$\vec{y}$ is a linear combination of these vectors when

$$\vec{y} = c_1\vec{v}_1 + c_2\vec{v}_2 \dots + c_p\vec{v}_p$$

for some scalars $c_1, c_2 \dots c_p$

find weights x_1, x_2, x_3 such that

$$x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{b}$$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} -4 \\ 3 \\ 8 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & -4x_2 & 2x_3 \\ 0 & 3x_2 & 5x_3 \\ -2x_1 & 8x_2 & -4x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{array}\right] \xrightarrow[2R_1+R_3]{} \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

$0 \neq 3$, so this system is ~~~~ inconsistent, which means that it has no solution.

$\therefore$ $\vec{b}$ is not a linear combination of the set of vectors $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$.

2. (3pts) List (at least) three distinct vectors living in Span$\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ where

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

The Span$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is the set of all linear combinations of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. If a vector lives in this span, that means it must be able to be written as a linear combination of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

You can obtain these vectors through scalar multiplication or vector addition. For example:

$3\vec{a}_1 = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$, $\vec{a}_1 + \vec{a}_3 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, $3\vec{a}_2 + \vec{a}_3 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, $2\vec{a}_1 - \vec{a}_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ all live in Span$\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$