Quiz 10

NOVEMBER 21, 2019

Name (Last, First)

Answer Key

1. Solve the initial value problem

$$\frac{d^2y}{dt^2} + y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

auxillary equation:

$r^2 + 1 = 0$

$r = \pm i$

for complex numbers,

$r = \alpha \pm Bi$

$y_1(t) = e^{\alpha t}\cos Bt$

$y_2(t) = e^{\alpha t}\sin Bt$

$y(t) = C_1 e^{\alpha t}\cos Bt + C_2 e^{\alpha t}\sin Bt$

from $r = \pm i$,

$\alpha = 0, B = 1$

$y(t) = C_1\cos t + C_2 \sin t \quad y(0) = C_1 = 1$

$y'(t) = -C_1 \sin t + C_2 \cos t \quad y'(0) = C_2 = 1$

so, $y = \cos t + \sin t$

↳ plug into givenequation to check

$\frac{d^2y}{dt^2} = -C_1\cos t - C_2 \sin t$

$-C_1\cos t - C_2\sin t + C_1 \cos t + C_2 \sin t = 0$ ✓

↓ works for when $C_1 = 1, C_2 = 1$

2. Solve the initial value problem

$$y'' - 5y' + 6y = -e^{2t}, \quad y(0) = 2, \quad y'(0) = 5.$$

auxillary equation:

$r^2 - 5r + 6 = 0$

$(r-2)(r-3) = 0$

$r = 2, 3$

↳ homogeneous solutions of $y_h = e^{2t}, e^{3t}$

to find all possible solutions,

$y_p'' - 5y_p' + 6y_p = -e^{2t}$

$-\ y'' - 5y' + 6y = -e^{2t}$

$(y-y_p)'' - 5(y-y_p)' + 6(y-y_p) = 0$

↳ $y - y_p = C_1 e^{2t} + C_2 e^{3t}$

$y = y_p + C_1 e^{2t} + C_2 e^{3t}$

$y(t) = te^{2t} + C_1 e^{2t} + C_2 e^{3t}$

$y'(t) = e^{2t} + 2te^{2t} + 2C_1 e^{2t} + 3C_2 e^{3t}$

$y(0) = C_1 + C_2 = 2$

$y'(0) = 1 + 2C_1 + 3C_2 = 5$

$C_1 = 2, C_2 = 0$

↳ $y(t) = te^{2t} + 2e^{2t}$

find the particular solution for the nonhomogeneous case

↳ $y = cte^{2t}$

↓ multiply by t b/c it matches homo.sol.

$y' = ce^{2t} + 2cte^{2t}$

$y'' = 2ce^{2t} + 2ce^{2t} + 4cte^{2t}$

$(2ce^{2t} + 2ce^{2t} + 4cte^{2t}) - 5(ce^{2t} + 2cte^{2t}) + 6(cte^{2t}) = e^{-2t}$

↳ $c = 1$

$y_p = te^{2t}$