

1. c, f
2. a
3. b
4. c
5. a (zero vector)
6. a, b, c
7. a
8. d
9. b
10. b
11. c
12. b, d
13. e
14. c
15. a
16. c
17. e
18. a, b, d, e
19. c
20. c
21. e
22. b
23. a, c
24. d
25. c
26. b
27. $\text{Span} \{[1, 1, 1, 0]^T, [0, 1, 1, 0]^T\}$
28. c
29. c
30. b
31. e
32. $\cos x \sim \cos x$
33. $|\sin x| \sim \frac{2}{\pi} + \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\cos 2kx}{1-4k^2}$.
 - a. For $x = 0$, we have $\frac{1}{2} = \sum_{k=1}^{\infty} \frac{-1}{1-4k^2}$.
 - b. For $x = \frac{\pi}{2}$, we have $\pi = 2 + 4 \sum_{k=1}^{\infty} \frac{(-1)^k}{1-4k^2}$.
34. $|\cos x| \sim \frac{2}{\pi} + \frac{2}{\pi} \sum_{k=1}^{\infty} \left(\frac{(-1)^k}{1+2k} + \frac{(-1)^k}{1-2k} \right) \cos 2kx$
35. $(\cos x)^2 \sim \frac{1}{2} + \frac{1}{2} \cos 2x$

$$36. (\sin x)^2 \sim \frac{1}{2} - \frac{1}{2} \sin 2x$$

$$37. |\sin x \cos x| \sim \frac{1}{\pi} - \sum_{k=1}^{\infty} \frac{2}{\pi(4k^2-1)} \cos 4kx$$

$$38. \sin x \sim \frac{2}{\pi} + \frac{1}{\pi} \sum_{k=1}^{\infty} \left(\frac{2}{1+2k} + \frac{2}{1-2k} \right) \cos 2kx \quad \{0 \leq x \leq \pi\}.$$

$$a. \text{ For } x = \frac{\pi}{2}, \text{ we have } \pi = 2 + 4 \sum_{k=1}^{\infty} \frac{(-1)^k}{1-4k^2}.$$

39. The method of “variation of parameters” is used to find a particular solution to the nonhomogeneous equation $y''(t) + a_1(t)y'(t) + a_0(t) = g(t)$.

After finding two solutions y_1 and y_2 to the homogeneous equation $y''(t) + a_1(t)y'(t) + a_0(t) = 0$, set up the following system of equations and

solve for v_1' and v_2' : $\begin{cases} v_1'y_1 + v_2'y_2 = 0 \\ v_1'y_1' + v_2'y_2' = g(t) \end{cases}$. Then, integrate to find v_1

and v_2 . A particular solution to the nonhomogeneous equation is

$$y_p = v_1y_1 + v_2y_2.$$

$$40. y(t) = \begin{cases} 1 + \frac{6}{e} e^{\cos t} & \{0 \leq t \leq \pi\} \\ -1 + \left(2e + \frac{6}{e}\right) e^{\cos t} & \{\pi \leq t \leq 2\pi\} \end{cases}$$

$$41. y = c \sec x - \frac{\cos 175x}{175 \cos x}, \text{ where } c \text{ is any constant}$$

$$42. \text{ Maximum displacement is } \frac{\sqrt{2}}{e^\pi}$$

$$43. a = -2, b = 5, f(t) = 4e^{2t} \sin t + 2e^{2t} \cos t + 5t - 2$$

$$44. x^4 \ln x$$

45. Match the graphs:

a. 2

b. 1

c. 4

d. 3

$$46. y = 8e^{7t} - 49te^{7t}$$

$$47. \mathbf{x}'(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & \frac{-1}{1+t} \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 0 \\ \sin t \end{bmatrix}$$

$$48. y = c_1 \cos 4x + c_2 \sin 4x + \frac{1}{4} x \sin 4x + \frac{1}{16} \ln |\cos 4x| \cos 4x$$

$$49. \mathbf{x}(t) = \begin{bmatrix} 5e^{8t} - 2e^{-8t} \\ -2e^{-8t} \end{bmatrix}$$

$$50. \mathbf{y}(t) = \begin{bmatrix} 6 \cos t + 8 \sin t \\ -5 \cos t - 15 \sin t \end{bmatrix}$$

51. $\mathbf{v}_1 \xi_1' + \mathbf{v}_2 \xi_2' = 1 \mathbf{v}_1 \xi_1 + 2\mathbf{v}_2 \xi_2 + \mathbf{v}_1(-2 \sin t - 3e^t) + \mathbf{v}_2(\sin t + e^t)$, where $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Then solve for ξ_1 and ξ_2 .

52. Decide if the statements are *always true* or *sometimes false*.

- a. F
- b. T
- c. F
- d. T
- e. T
- f. T
- g. T
- h. F
- i. T
- j. F
- k. T