

Name (Last, First)

1. (5pts) Solve the following differential equations.

a.

$$y''(t) - 8y'(t) + 16y(t) = 0, \quad y(1) = 3, \quad y'(1) = 13$$

b.

$$z''(x) + z(x) = 2e^{-x}, \quad z(0) = 0, \quad z'(0) = 0$$

2. (5pts) In this problem, we want to prove a formula of sines and cosines.

a. Solve the initial value problem $y''(t) + y(t) = 0$, $y(0) = \sin 9^\circ$, and $y'(0) = \cos 9^\circ$.

b. Let $f(t) := \sin(9^\circ + t)$. Show that $f''(t) + f(t) = 0$, $f(0) = \sin 9^\circ$, and $f'(0) = \cos 9^\circ$.

c. What can you say about $y(t)$ from a and $f(t)$ from b?¹ Write down the formula $y(t) = f(t)$ explicitly.

¹In homogeneous case, any initial value problem has a unique solution.