

Name (Last, First)

1. (3pts) Find a singular value decomposition of A .

$$A = \begin{bmatrix} 3 & -3 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

2. (7pts) Let a symmetric matrix A be the following:

$$A = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}.$$

a. Check if $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector and find the corresponding eigenvalue.

b. It is known that 5 is an eigenvalue of A . Find all eigenvalues and orthogonally diagonalization of A .

c. Let $Q(x, y, z) = 4x^2 + 4y^2 + 4z^2 - 2xy - 2yz - 2xz$. Express $Q(x, y, z)$ as the sum of 3 weighted squares. In other words, transform it into another one with no cross-product term.¹

¹For example, $-x^2 + y^2 - z^2 + 2xy + 4xz = (x + y)^2 - 2(x - z)^2 + z^2$. **Hint** : Use the spectral theorem.