Quiz 6

October 11, 2018

Name (Last, First)

1. (5pts) Find the orthogonal projection of $\mathbf{x}$ onto $\mathbf{u}_1$ and $\mathbf{u}_2$ respectively.

$$\mathbf{x} = \begin{bmatrix} 7 \\ 9 \\ 1 \\ 0 \end{bmatrix}, \ \mathbf{u}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \ \mathbf{u}_2 = \begin{bmatrix} 3 \\ -3 \\ 3 \\ -3 \end{bmatrix}$$

2. (3pts) Let W be a subspace of $\mathbb{R}^3$ with a basis

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}.$$

Let A be a 3×2 matrix given by

$$\begin{bmatrix} 1 & 0 & 3 \\ 1 & 2 & 0 \end{bmatrix}.$$

a. What is Nul A? Find a basis.

b. What is $W^{\perp}$? Find a basis.

c. What is $\dim W + \dim W^{\perp}$? Check if it is the same as $\text{rk}A + \dim \text{Nul}A$.

3. (2pts) Let W be a subspace of $\mathbb{R}^n$, and let $W^{\perp}$ be the set of all vectors orthogonal to W. Show that $W^{\perp}$ is a subspace[1] of $\mathbb{R}^n$.

***Caution.** Please be as explicit as possible. Write down as if you are explaining to someone else. Only numbers or variables (such as x, y, v, etc.) without any explanation will not give you enough credits. You can use words like 'if', 'then', 'suppose', 'let', 'therefore', etc.

[1] $\mathbf{0}$, closed under addition, and closed under scalar multiplication!