Student ID Name (Last, First)

1. Let T be a linear transformation satisfying

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y \\ z \\ x+y+z \end{bmatrix}.$$

a Find the standard matrix, say A, of T and find any[1] **nonzero** vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{0}$.

b Explain why T is not **one-to-one** by comparing:

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) \quad \text{and} \quad T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \mathbf{x}\right)$$

[1] Yes, this means you do NOT need to find the whole null space of A in this problem. Just find one of them.

2. Prove why this statement is true : A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ is completely determined by its effect on the columns of the $n \times n$ identity matrix.

***Caution.** Please be as explicit as possible. Write down as if you are explaining to someone else. Only numbers or variables (such as x, y, v, etc.) without any explanation will not give you enough credits.