Name:

MATH 54 QUIZ 2

THU, SEP 6, 2018

Time: 15 minutes.

(1) Find an explicit description of Nul A by listing vectors that span the null space:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Solution. The null space of A is exactly the same as the null space of any row reduction of A. So, replace 2nd row by 2nd row - 1st row:

$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix}$, and then divide 2nd row by 3: $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$.

Now, replacy 1st row by 1st row - 2nd row:

$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$. Switch 1st and 2nd rows: $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. So, the RREF of A would be: $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$.

Now, the null space of the last matrix consists of

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

satisfying $x_1 - x_3 = 0$ and $x_2 + 2x_3 = 0$, that means, $x_1 = x_3$ and $x_2 = -2x_3$ and x_3 can have any value, that is,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.$$

So, the null space of A is spanned by $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

(2) Find the pivot columns of A and compute the dimension of Col A:

$$A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$$

Solution. Let's find the RREF of A. Divide 3rd row by 4 and then switch the position with 1st row:

$$\begin{bmatrix} 1 & 0 & 1 \\ 6 & 4 & 8 \\ -8 & -2 & -9 \end{bmatrix}.$$

Now, replace 2nd row by 2nd row - 6× 1st row and replace 3rd row by 3rd row + 8× 1st row:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 4 & 2 \\ 0 & -2 & -1 \end{bmatrix}.$$

Replace 2nd row by 2nd row + 2× 3rd row and switch 2nd row and 3rd row:

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \text{ so that the RREF would be } \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0.5 \\ 0 & 0 & 0 \end{bmatrix}.$$

Hence, the pivot columns of A is the 1st and 2nd columns of A, that is, $\begin{bmatrix} -8 \\ 6 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}$. By a theorem, pivot columns form a basis for Col A. So, in this case, the dimension of Col A would be 2.