Quiz 7

1. Find the dimension of the subspace H inside of $\mathbb{R}^3$ given by all vectors of the form

$$\begin{bmatrix} a+b+c \\ b+d \\ a+b+d \end{bmatrix}$$

where a, b, c, d are any real numbers.

$\begin{bmatrix} a+b+c \\ b+d \\ a+b+d \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Hence, H is the column space of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}.$$

Here, let's do row reduction for A so that we get

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}.$$

Hence, we have 3 pivots and, as a result, Col $A = \mathbb{R}^3$. So, H is $\mathbb{R}^3$ so that H has dimension $\mathbf{3}$.

2. If A is a 5×4 matrix, what is the largest possible dimension of the row space of A? What is the largest possible dimension of the null space Nul A?

Note that row vectors live in $\mathbb{R}^4$. Hence, the largest possible dimension of the row space of A is $\mathbf{4}$. Also, the largest possible dimension of the null space is also $\mathbf{4}$ because Nul A is a subspace of $\mathbb{R}^4$.

Examples[1] for each cases can be

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

[1] To give more correct, detailed, and enough explanation, you should give an example for each case. Even though I did not take this into account for grading this time, please try to give such an example next time.

1. Find the dimension of the subspace H inside of $\mathbb{R}^3$ given by all vectors of the form

$$\begin{bmatrix} a+b+2c \\ -a+b \\ b+c \end{bmatrix}$$

where a, b, c are any real numbers.

Note that $\begin{bmatrix} a+b+2c \\ -a+b \\ b+c \end{bmatrix} = a\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$. Hence, H is the same as the column space of

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

Applying row reduction processes, we get

$$\begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

Hence, there are two pivots so that Col A has dimension $\mathbf{2}$ with a basis $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

Therefore, the dimension of H is $\mathbf{2}$.

2. If A is a 3×5 matrix, what is the largest possible dimension of the row space of A? What is the largest possible dimension of the null space $\mathrm{Nul}\ A$?

The row space of A is the same as the column space of A^T which has 5×3 size. Since there are only three columns in A^T, there could be at most 3 pivot columns so that the largest possible dimension of the column space of A^T is $\mathbf{3}$. Hence, the largest possible dimension of the row space of A is $\mathbf{3}$. Next, for $\mathrm{Nul}\ A$, because $\mathrm{Nul}\ A$ lives in $\mathbb{R}^5$, the largest possible dimension of $\mathrm{Nul}\ A$ is $\mathbf{5}$.

Examples[2] for each cases can be

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

[2] To give more correct explanation, you should give an example for each case. Even though I did not take this into account for grading this time, please try to give such an example next time.