Quiz 6

Name : ____________________

SID : ____________________

1. Let a subset W of $M_{2\times 2}$ be given as

$$W = \left\{ A \in M_{2\times 2} | A^2 = \mathbf{0} \right\}$$

where $\mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Is W a subspace of $M_{2\times 2}$? Why or why not?

2. Let $\mathcal{B} = \left\{ b_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, b_2 = \begin{pmatrix} 2 \\ 3 \\ -3 \end{pmatrix}, b_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$ be a basis for $\mathbb{R}^3$.

a) Let

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},\ e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},\ e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

We are given that

$$\begin{aligned} e_1 &= 3b_1 - b_2 + 3b_3 \\ e_2 &= b_3 \\ e_3 &= 2b_1 - b_2 + 3b_3 \end{aligned}$$

Find the inverse matrix $P_{\mathcal{B}}^{-1}$ and explain why. (You can use the formula $v = P_{\mathcal{B}} \cdot [v]_{\mathcal{B}}$.)

b) Let $u = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. Find $[u]_{\mathcal{B}}$.