Quiz 5, MATH 54, Fall 2014

Name (Last, First): ______________________

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(1) Let A be a 3×4 matrix given as

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 5 & 8 & 11 \\ 1 & 3 & 6 & 10 \end{bmatrix}.$$

What is the rank of A? Is Nul $A = \mathbb{R}^4$? Is Col $A = \mathbb{R}^3$?

$$\begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 5 & 8 & 11 \\ 1 & 3 & 6 & 10 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

There are 3 pivots $\Rightarrow$ the rank of $A = 3$.

Since every row has a pivot, Col $A = \mathbb{R}^3$.

By the rank theorem, dim Nul $A = 4 - 3 = 1$.

So, Nul $A \neq \mathbb{R}^4$ since dim $\mathbb{R}^4 = 4 \neq 1$

(2) Compute the determinant of the following matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 5 & 2 & 2 & 3 & 3 & 0 \\ 51 & 0 & 0 & 1 & 1 & 0 \\ 3 & 1 & -1 & 1 & 2 & 0 \\ 10 & -1 & 3 & 1 & 2 & 1 \\ 127 & 0 & 1 & 3 & 2 & 7 \end{bmatrix}$$

Using cofactor expansion with the first row we only need to compute $\det A_{11}$.
~~By~~ Let's change 5^{th} row by subtracting 7×4^{th} row. and then use
cofactor expansion using fifth column.
So, the determinant would be (-1) times

$$\det\begin{pmatrix} 2 & 2 & 3 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 7 & -20 & -4 & -12 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 2 & 3 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 7 & -20 & -4 & -12 \end{pmatrix} \underset{R}{\sim} \begin{pmatrix} 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 7 & -20 & -4 & -12 \end{pmatrix} \overset{2}{\underset{R}{\sim}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & -2 & 1 & 2 \\ 0 & -27 & -4 & -12 \end{pmatrix} \overset{(-1)}{\underset{R}{\sim}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -17.5 & -38 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & & & * \\ & -2 & & \\ & & 1 & \\ 0 & & & -21.5 \end{pmatrix}$$

Hence the result is

$$(-1) \cdot 2 \cdot (-1) \cdot (-2) \cdot (-21.5) = 86$$