Quiz 5, MATH 54, Fall 2014

Name (Last, First): _____

Student ID: _____

(1) Let $A$ be a $3 \times 7$ matrix given as

$$\begin{bmatrix} 1 & 3 & 5 & 7 & 9 & 11 & 13 \\ 2 & 5 & 8 & 11 & 14 & 17 & 20 \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 \end{bmatrix}.$$

What is the rank of $A$? Is Nul $A = \mathbb{R}^4$? Is Col $A = \mathbb{R}^3$?

$$\begin{pmatrix} 1 & 3 & 5 & 7 & 9 & 11 & 13 \\ 2 & 5 & 8 & 11 & 14 & 17 & 20 \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 & 11 & 13 \\ 0 & -1 & -2 & -3 & -4 & -5 & -6 \\ 0 & 0 & 1 & 3 & 6 & 10 & 15 \end{pmatrix}$$

There are 3 pivots $\Rightarrow$ the rank of $A = 3$.

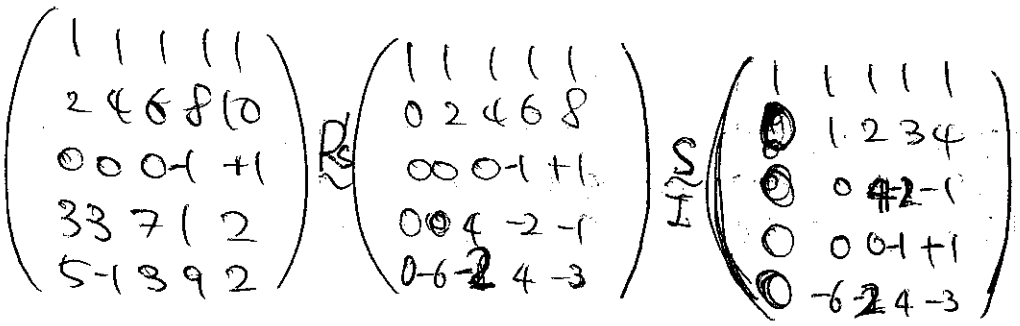
Since $A$ has a pivot in every row, Column vectors span $\mathbb{R}^3$.

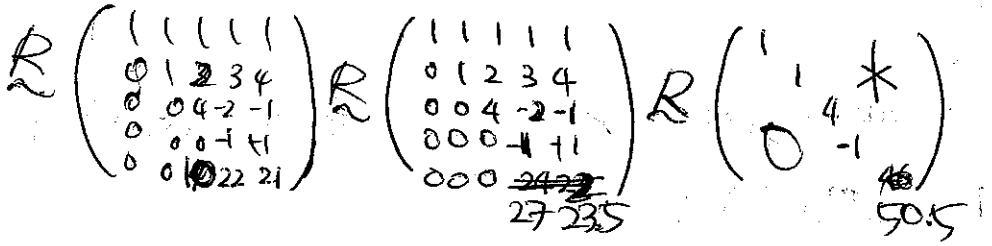
$\Rightarrow$ Col $A = \mathbb{R}^3$.

However, Nul $A \neq \mathbb{R}^4$ b/c Nul $A \subseteq \mathbb{R}^7$ but $\mathbb{R}^4 \not\subseteq \mathbb{R}^7$.

(2) Compute the determinant of the following matrix.

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 & 10 \\ 0 & 0 & 0 & -1 & 1 \\ 3 & 3 & 7 & 1 & 2 \\ 5 & -1 & 3 & 9 & 2 \end{bmatrix}$$





There is one Scaling & one Interchange.

$$\Rightarrow \det M = 2 \cdot (-1) \cdot 1 \cdot 1 \cdot 4 \cdot (-1) \cdot 50.5$$

$$= 404.$$