

Name (Last, First): _____

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(1) Let A be a 3×7 matrix given as

$$\begin{bmatrix} 1 & 3 & 5 & 7 & 9 & 11 & 13 \\ 2 & 5 & 8 & 11 & 14 & 17 & 20 \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 \end{bmatrix}$$

What is the rank of A ? Is $\text{Nul } A = \mathbb{R}^7$? Is $\text{Col } A = \mathbb{R}^3$?

$$\begin{pmatrix} 1 & 3 & 5 & 7 & 9 & 11 & 13 \\ 2 & 5 & 8 & 11 & 14 & 17 & 20 \\ 1 & 3 & 6 & 10 & 15 & 21 & 28 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 5 & 7 & 9 & 11 & 13 \\ 0 & -1 & -2 & -3 & -4 & -5 & -6 \\ 0 & 0 & 1 & 3 & 6 & 10 & 15 \end{pmatrix}$$

There are 3 pivots \Rightarrow the rank of $A = 3$.

Since A has a pivot in every row, Column vectors span \mathbb{R}^3 .

$$\Rightarrow \text{Col } A = \mathbb{R}^3.$$

However, $\text{Nul } A \neq \mathbb{R}^7$ b/c ~~\mathbb{R}^7~~ $\text{Nul } A \subseteq \mathbb{R}^7$ but

$$\mathbb{R}^4 \not\subseteq \mathbb{R}^7.$$

(2) Compute the determinant of the following matrix.

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 & 10 \\ 0 & 0 & 0 & -1 & 1 \\ 3 & 3 & 7 & 1 & 2 \\ 5 & -1 & 3 & 9 & 2 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 & 10 \\ 0 & 0 & 0 & -1 & 1 \\ 3 & 3 & 7 & 1 & 2 \\ 5 & -1 & 3 & 9 & 2 \end{pmatrix} \xrightarrow{R_2} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 8 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 4 & -2 & -1 \\ 0 & -6 & 2 & 4 & -3 \end{pmatrix} \xrightarrow{R_3} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 8 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 4 & -2 & -1 \\ 0 & -6 & 2 & 4 & -3 \end{pmatrix}$$

$$\xrightarrow{R_4} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 8 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 2 & 2 & 2 \end{pmatrix} \xrightarrow{R_5} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 8 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 2 & 2 \end{pmatrix} \xrightarrow{R_6} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 8 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

There is one Scaling & one Interchange.

$$\Rightarrow \det M = 2 \cdot (-1) \cdot 1 \cdot 1 \cdot 4 \cdot (-1) \cdot \cancel{50.5} = 404.$$