Quiz 4, MATH 54, Fall 2014

Name (Last, First): ______________________________

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1. Suppose $A = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}$ is the standard matrix for a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ and $B = \begin{bmatrix} 6 & 7 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$ is the standard matrix for a linear function $S : \mathbb{R}^2 \to \mathbb{R}^4$.

   a) Check if A is an invertible matrix. (If it is, find the inverse. If not, prove why it is not invertible.)

$ad-bc \neq 0$ for A.

So, $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$= \frac{1}{1}\begin{pmatrix} -1 & -1 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ -1 & -2 \end{pmatrix}$.

   b) Find the standard matrix for $S \circ T \circ T^{-1} \circ T$.

$T \circ T^{-1} = Id \Rightarrow$ the matrix will be $B \cdot A$.

$$\begin{pmatrix} 6 & 7 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} -5 & -1 \\ 0 & 0 \\ -2 & 1 \\ 0 & 0 \end{pmatrix}$$

2. Let $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$. Find a basis for Col A and a basis for Nul A.

$$A \sim \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ 1st, 2nd column has a pivot.

$$\Rightarrow \text{A basis for Col } A = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$\text{Nul } A = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \middle| \; x_1 = x_3,\; x_2 = x_4 \right\}$$

$$= \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} x_2 \;\middle|\; x_1, x_2 \in \mathbb{R} \right\}$$

$$= \text{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

$$\Rightarrow \text{A basis for Nul } A = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$