Name (Last, First): ______________________________

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1. Suppose $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is the standard matrix for a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 4 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$ is the standard matrix for a linear function $S : \mathbb{R}^2 \to \mathbb{R}^4$.

   a) Check if A is an invertible matrix. (If it is, find the inverse. If not, prove why it is not invertible.)

   ad−bc of A is $1 \neq 0$.

   $$\therefore A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

   b) Find the standard matrix for $S \circ T \circ T \circ T$.

   $$\begin{pmatrix} 1 & 0 \\ 3 & 4 \\ -1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3 & 4 \\ -1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

   $$= \begin{pmatrix} 1 & 0 \\ 3 & 4 \\ -1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

   $$= \begin{pmatrix} 1 & 3 \\ 3 & 13 \\ -1 & -3 \\ 0 & 0 \end{pmatrix}.$$

2. Let $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$. Find a basis for Col A and a basis for Nul A.

$$A \approx \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \sim \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{RREF}$$: 1st, 2nd, 4th columns have pivots.

$$\Rightarrow \text{A basis for Col } A = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}.$$

From the RREF, we know that

$$\text{Nul } A = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \middle| \ x_1 = 0,\ x_2 = x_3,\ x_4 = 0 \right\}.$$

$$= \left\{ x_2 \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \middle| \ x_2 \in \mathbb{R} \right\}$$

$$\therefore \text{A basis for Nul } A = \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\}.$$