

1. Use Stokes' Theorem to evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$ .  $C$  is oriented counterclockwise as viewed from above.

$$\mathbf{F}(x, y, z) = xy\mathbf{i} + 2z\mathbf{j} + 3y\mathbf{k},$$

$C$  is the curve of intersection of the plane  $x + z = 5$  and the cylinder  $x^2 + y^2 = 9$ .

2. Suppose  $S$  and  $C$  satisfy the hypotheses of Stokes' Theorem and  $f, g$  have continuous second-order partial derivatives. Show that

a)

$$\int_C (f\nabla g) \cdot d\mathbf{r} = \iint_S (\nabla f \times \nabla g) \cdot d\mathbf{S}$$

b) As a corollary of a),

$$\int_C (f\nabla g + g\nabla f) \cdot d\mathbf{r} = 0$$

3. Use the Divergence Theorem to calculate the surface integral  $\iint_S \mathbf{F} \cdot d\mathbf{S}$ ; that is, calculate the flux of  $\mathbf{F}$  across  $\mathbf{S}$ .

(a)

$$\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + xy^2z\mathbf{j} + xyz^2\mathbf{k},$$

where  $S$  is the surface of the box enclosed by the planes  $x = 0$ ,  $x = a$ ,  $y = 0$ ,  $y = b$ ,  $z = 0$ , and  $z = c$ , where  $a$ ,  $b$ , and  $c$  are positive numbers.

(b)

$$\mathbf{F}(x, y, z) = x^2 \sin y\mathbf{i} + x \cos y\mathbf{j} - xz \sin y\mathbf{k},$$

where  $S$  is the “fat sphere”  $x^8 + y^8 + z^8 = 8$ .

(c)

$$\mathbf{F}(x, y, z) = (\cos z + xy^2)\mathbf{i} + xe^{-z}\mathbf{j} + (\sin y + x^2z)\mathbf{k},$$

where  $S$  is the surface of the solid bounded by the paraboloid  $z = x^2 + y^2$  and the plane  $z = 4$ .

Course (the last) Homework due Review session (TBD)

Apr 28, Mon. : **16.7** 19, 21, 23, 27. **16.8** 1, 3, 5

Apr 30, Wed. : **16.8** 7, 9, 11(a), 13, 15, 19, 20

May 2, Fri. : **16.9** 1, 3, 5, 7, 9, 11, 13, 17, 23, 24, 25, 26, 27