Solution 6

1. Use the Chain Rule to find dz/dt for a) and $\partial z/\partial s$, $\partial z/\partial t$ for b).

a) $z = \dfrac{x^3 - x\ln y + y}{\sin y}, \quad x = e^t,\ y = t^2$

Answer. $\dfrac{e^t(3e^{2t} - 2\ln t)}{\sin(t^2)}$
$+2t\dfrac{\sin(t^2)(1-\frac{e^t}{t^2}) - \cos(t^2)(e^{3t} - 2e^t\ln t + t^2)}{\sin^2(t^2)}$

b) $z = x\sin\theta, \quad x = \dfrac{s}{t},\ \theta = s^2 + t$

Answer. $\dfrac{\partial z}{\partial s} = \dfrac{1}{t}\sin(s^2+t) + \dfrac{2s^2}{t}\cos(s^2+t)$
$\dfrac{\partial z}{\partial t} = -\dfrac{s}{t^2}\sin(s^2+t) + \dfrac{s}{t}\cos(s^2+t)$

2. Find dy/dx for a) and $\partial z/\partial x$, $\partial z/\partial y$ for b) and c).

a) $x^2 + \sin x\sin y - y^2 = 0$

Solution. Using the formula for implicit differentiation

$$\frac{dy}{dx} = -\frac{F_x}{F_y},$$

we get the answer.

Answer. $\dfrac{dy}{dx} = \dfrac{2x + \cos x\sin y}{2y - \sin x\cos y}$

b) $x + y^2 + z^3 = 0$

Solution. Using the formula for implicit differentiation

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \quad \& \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z},$$

we get the answer.

Answer. $\dfrac{\partial z}{\partial x} = -\dfrac{1}{3z^2}$, $\dfrac{\partial z}{\partial y} = -\dfrac{2y}{3z^2}$

c) $\tan x + e^y + z^3 - z^2 = 0$

Solution. Same as before.

Answer. $\dfrac{\partial z}{\partial x} = -\dfrac{1}{z(3z-2)\cos^2 x}, \ \dfrac{\partial z}{\partial y} = -\dfrac{e^y}{z(3z-2)}$

3. Find the gradient of f, evaluate the gradient at the point P, and find the rate of change of f at P in the direction of the vector $\mathbf{u}$.

$$f(x, y, z) = \cos(x^2) + xy + \ln z, \quad P = (\pi, 1, e) \quad \mathbf{u} = \left\langle \frac{1}{9}, -\frac{8}{9}, \frac{4}{9} \right\rangle$$

Solution. The gradient of f is defined as following ;

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right).$$

Hence, in this problem, $\nabla f = (-2x\sin(x^2) + y, x, \frac{1}{z})$. At the point P,

$$\nabla f(P) = (-2\pi \sin(\pi^2) + 1, \pi, \tfrac{1}{e}).$$

The rate of change of f at P in the direction of the vector $\mathbf{u}$ is given as

$$\nabla f(P) \cdot \mathbf{u} = \frac{1}{9} \left(-2\pi \sin(\pi^2) + 1 - 8\pi + \frac{4}{e} \right).$$

Answer. Answers are listed in the ***Solution***.

Letter grade for Quiz 6

$A^+ = 20$	(6)
$A0 = 19$	(6)
$B^+ = 18$	(9)
$B0 = 16,\ 17$	(6)
$C^+ = ..$	(2)