1. Suppose $\mathbf{u}$ and $\mathbf{v}$ are differentiable vector functions, c is a scalar, and f is a real-valued function. Prove

$$\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{v}(t)] = \mathbf{u}'(t) \times \mathbf{v}(t) + \mathbf{u}(t) \times \mathbf{v}'(t), \qquad \frac{d}{dt}[\mathbf{u}(f(t))] = f'(t)\mathbf{u}'(f(t)).$$

2. Find the differential of the function T.

$$T = \frac{v}{1 + uvw}$$

3. Assume that all the given functions are differentiable.

   a) If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2.$$

   b) If $z = f(x, y)$, where $x = s + t$ and $y = s - t$, show that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = \frac{\partial z}{\partial s}\frac{\partial z}{\partial t}.$$

4. Suppose that the equation $F(x, y, z) = 0$ implicitly defines each of the three variables x, y, and z as functions of the other two: $z = f(x, y)$, $y = g(x, z)$, $x = h(y, z)$. If F is differentiable and F_x, F_y, and F_z are all nonzero, show that

$$\frac{\partial z}{\partial x}\frac{\partial x}{\partial y}\frac{\partial y}{\partial z} = -1.$$

Course Homework due Mar 19, Wed.

Mar 10, Mon. : **15.3** 1, 3, 5, 7, 9, 11, 19, 21, 39, 41
Mar 12, Wed. : **15.4** 7, 9, 11, 13, 15, 17, 21, 23, 29
Mar 14, Fri. : **Midterm #1**