

Differentiation

Basic Functions

$$\frac{d}{dx} x^r = r \cdot x^{r-1} \quad (r : \text{a real number})$$

$$\frac{d}{dx} \sin x = \cos x \quad \frac{d}{dx} \cos x = -\sin x \quad \frac{d}{dx} e^x = e^x \quad \frac{d}{dx} \ln x = \frac{1}{x}$$

Remembering only three things, you can differentiate almost all functions given in this course.

$$\begin{aligned} \frac{d(f(x) \cdot g(x))}{dx} &= f(x) \cdot \frac{d(g(x))}{dx} + \frac{d(f(x))}{dx} \cdot g(x) & (f(x)g(x))' &= f'(x)g(x) + f(x)g'(x) \\ \frac{d\left(\frac{f(x)}{g(x)}\right)}{dx} &= \frac{\frac{d(f(x))}{dx} \cdot g(x) - f(x) \cdot \frac{d(g(x))}{dx}}{g(x)^2} & \left(\frac{f(x)}{g(x)}\right)' &= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \\ \frac{d((f \circ g)(x))}{dx} &= \frac{d(f(x))}{dx}(g(x)) \cdot \frac{d(g(x))}{dx} & (f(g(x)))' &= f'(g(x)) \cdot g'(x) \end{aligned}$$

Exercise.

- | | | | |
|---------------------------|-----------------------------------|-------------------|----------------------------|
| 1) $\sin 2x$ | 2) $\cos 2x^4$ | 3) $\sec 5x^3$ | 4) $\tan 3x$ |
| 5) $2x^2 \sin x^3$ | 6) $e^{x^3+2x^2+x+1}$ | 7) $2x^2 e^{x^3}$ | 8) $\sin 3x \cos 5x^2$ |
| 9) $\frac{3 \sin 4x}{2x}$ | 10) $\frac{5e^{3x}}{3 \sin 2x^2}$ | 11) e^{x^2} | 12) $\frac{\ln x}{\sin x}$ |

Integration

Differently with differentiation, in general, it is pretty hard to integrate a function even though you might know many useful formulas. I would introduce how to integrate specific functions every time they appear in homework sets or exercises from the textbook.

Here, I want to note two useful formulas for calculating integration.

$$\begin{aligned} \int_{\alpha}^{\beta} f'(x)g(x)dx &= \left(f(\beta)g(\beta) - f(\alpha)g(\alpha) \right) - \int_{\alpha}^{\beta} f(x)g'(x)dx \\ \int_{\alpha}^{\beta} f(x)dx &= \int_{p^{-1}(\alpha)}^{p^{-1}(\beta)} f(p(t)) \cdot p'(t)dt \quad (\text{where } x = p(t)) \end{aligned}$$

Exercise.

- | | | | | |
|--------------------------|----------------------------|------------------------|---|--------------------------------|
| 1) $\int_0^5 \sin 3x dx$ | 2) $\int_0^3 x e^{x^2} dx$ | 3) $\int_0^2 x e^x dx$ | 4) $\int_0^{\sqrt{3}} \frac{1}{1+x^2} dx$ | 5) $\int_0^1 \sqrt{1+4x^2} dx$ |
|--------------------------|----------------------------|------------------------|---|--------------------------------|

Course Homework due Feb 5, Wed.

Jan 27, Mon. : **10.3** 15, 17, 21, 25, 56, 57. **10.4** 5, 7

Jan 29, Wed. : **10.4** 9, 11, 17, 45, 47. **10.5** 5, 11, 15, 19 (sketch graphs only)

Jan 31, Fri. : **12.1** 13, 15, 31. **12.2** 17, 21, 23. **12.3** 1, 7, 23, 37

Answer (Differentiation).

1) $2 \cos 2x$

2) $-8x^3 \sin 2x^4$

3) $15x^2 \frac{\sin 5x^3}{\cos^2 5x^3}$

4) $\frac{1}{\cos^2 3x}$

5) $4x \sin x^3 + 6x^4 \cos x^3$

6) $(3x^2 + 4x + 1)e^{x^3+2x^2+x+1}$

7) $(4x + 6x^4)e^{x^3}$

8) $3 \cos 3x \cos 5x^2 - 10x \sin 3x \sin 5x^2$

9) $\frac{3}{2} \frac{4x \cos x - \sin 4x}{x^2}$

10) $\frac{5}{3} \frac{3e^{3x} \sin 2x^2 - 4xe^{3x} \cos 2x^2}{\sin^2 2x^2}$

11) $2xe^{x^2}$

12) $\frac{\sin x - x \ln x \cos x}{x \sin^2 x}$

Answer (Integration).

1) $\frac{1}{3}(1 - \cos 15)$

2) $\frac{1}{2}(e^9 - 1)$

3) $e^2 + 1$

4) $\frac{\pi}{3}$

5) $\frac{1}{4}(2\sqrt{5} + \ln(2 + \sqrt{5}))$