Formulas

- **Trigonometric Functions**

  First of all, the definition of $\tan x$ is $\frac{\sin x}{\cos x}$. You have one thing that is very useful in many cases.

  $$\sin^2 x + \cos^2 x = 1$$

  Another formulas you need to know are

  $$\sin(x+y) = \sin x \cos y + \cos x \sin y \tag{1}$$
  $$\sin(x-y) = \sin x \cos y - \cos x \sin y \tag{2}$$
  $$\cos(x+y) = \cos x \cos y - \sin x \sin y \tag{3}$$
  $$\cos(x-y) = \cos x \cos y + \sin x \sin y \tag{4}$$

  Apparently, (2) can be induced from (1) and (4) can be derived from (3). The formula belows is the result of (1) and (3),

  $$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}.$$

  If we set $y = x$ in (1), (3), we get some useful formula,

  $$\sin 2x = 2 \sin x \cos x \tag{5}$$
  $$\cos 2x = \cos^2 x - \sin^2 x \tag{6}$$

  Using the fact that $\sin^2 x + \cos^2 x = 1$ holds, (6) have alternative versions (7) and (8),

  $$\cos 2x = 2\cos^2 x - 1 \tag{7}$$
  $$= 1 - 2\sin^2 x \tag{8}$$

  Moreover, $\frac{1}{2}\big((1) + (2)\big)$ gives

  $$\sin x \cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y)).$$

  In a very similar way, you get these formulas using (1)-(2), (3)+(4), and (3)-(4)

  $$\cos x \sin y = \frac{1}{2}(\sin(x+y) - \sin(x-y))$$
  $$\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$$
  $$\sin x \sin y = \frac{1}{2}(-\cos(x+y) + \cos(x-y))$$

- **Exponential Functions**

  For exponential functions, you need to know just one thing

  $$e^a \times e^b = e^{a+b}. \tag{9}$$

- **Logarithm Functions**

  Along with exponential functions, be aware of the meaning of $\ln x$. It is the number satisfying

  $$e^{\ln x} = x.$$

  Basically, the definition of a^n is $e^{n \times \ln a}$ for any positive real number a. For the next, using (9), you get

  $$\ln a + \ln b = \ln(a \times b)$$
  $$-\ln a = \ln\left(\frac{1}{a}\right)$$