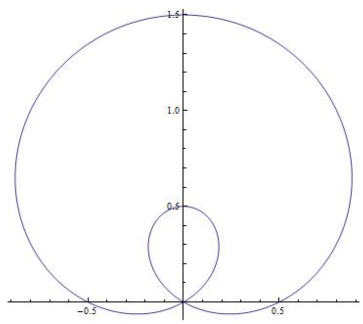
SOLUTION 2

1. Find the slope of the tangent line to the given polar curve at the point specified by the value of θ.

$$r = \sin\theta + 0.5, \qquad \theta = \frac{5\pi}{3}$$



Solution. We can change (r, θ)-coordinate to (x, y)-coordinate by setting $x = r\cos\theta$, $y = r\sin\theta$. Hence,

$$x = (\sin\theta + \tfrac{1}{2})\cos\theta,$$
$$y = (\sin\theta + \tfrac{1}{2})\sin\theta.$$

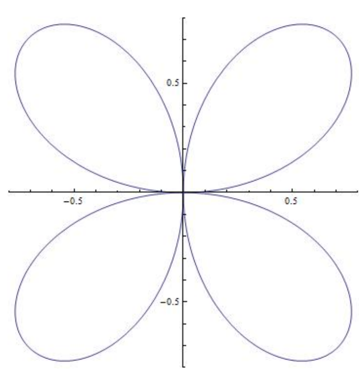
Now, we can get the slope of the tangent line as following ;

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\sin\theta\cos\theta + \frac{1}{2}\cos\theta}{\cos^2\theta - \sin^2\theta - \frac{1}{2}\sin\theta}.$$

Note that $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$ and $\cos\frac{5\pi}{3} = \frac{1}{2}$. So, we have the answer $4 + 3\sqrt{3}$.

Answer. $4 + 3\sqrt{3}$.

2. Find the area of 4 leaves of the graph of $r = \sin 2\theta$.



Solution. Let's use the formula

$$A = \frac{1}{2}\int_{\theta_0}^{\theta_1} r^2 d\theta.$$

We may first calculate the area of 1 leaf and then multiply 4. When θ varies from 0 to $\frac{\pi}{2}$, (r, θ) draws one leaf. Now, we need to compute

$$\begin{aligned} 4 \times \frac{1}{2}\int_0^{\frac{\pi}{2}} r^2 d\theta &= 2\int_0^{\frac{\pi}{2}} \sin^2 2\theta d\theta \\ &= \int_0^{\frac{\pi}{2}} (1 - \cos 4\theta) d\theta \\ &= \frac{\pi}{2} - \frac{1}{4}(\sin 2\pi - \sin 0) = \frac{\pi}{2}. \end{aligned}$$

Answer. $\frac{\pi}{2}$.

3. Determine whether the given vectors are orthogonal, parallel, or neither.

a) $\mathbf{a} = \langle -3, 2, 7\rangle$, $\mathbf{b} = \langle 2, 3, 0\rangle$
Solution. $\mathbf{a} \cdot \mathbf{b} = -6 + 6 + 0 = 0$. $\therefore$ Orthogonal.

b) $\mathbf{a} = \langle 3, -5\rangle$, $\mathbf{b} = \langle -9, 15\rangle$
Solution. $\mathbf{b} = -3\mathbf{a}$. $\therefore$ Parallel.

c) $\mathbf{a} = \mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 4\mathbf{j} + 13\mathbf{k}$
Solution. $\mathbf{a} \cdot \mathbf{b} = -1 + 28 + 26 = 53 \neq 0$. $\therefore$ Not Orthogonal. $|\mathbf{a}| \cdot |\mathbf{b}| = \sqrt{54}\sqrt{186} \neq \pm 53$. $\therefore$ Not Parallel.

d) $\mathbf{a} = -2\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} - 12\mathbf{j} + 6\mathbf{k}$
Solution. $3\mathbf{a} = -2\mathbf{b}$. $\therefore$ Parallel.

Answer. a) Orthogonal b) Parallel
c) Neither d) Parallel

Letter grade for Quiz 2

$$\begin{aligned} 29.0 &< A^+ \\ 27.0 &< A0 \leq 29.0 \\ 24.0 &< A^- \leq 27.0 \\ 23.0 &< B^+ \leq 24.0 \\ 22.0 &< B0 \leq 23.0 \\ 18.5 &< B^- \leq 22.0 \\ 10.0 &< C^+ \leq 18.5 \\ & C0 \leq 10.0 \end{aligned}$$