1. Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$.

$$z = x^2 + \sin y, \quad x = st, \quad y = \sin(s+t)$$

**A.** $\partial z/\partial s = 2st^2 + \cos(\sin(s+t))\cos(s+t)$, $\partial z/\partial t = 2s^2t + \cos(\sin(s+t))\cos(s+t)$

# Implicit Differentiation

2. Find $dy/dx$.

a) $x^2 - y^2 = xy$ b) $\cos x \sin y = xy$ c) $\cos(xy) = 1 + \sin y$

**A.** a) $\frac{2x-y}{x+2y}$ b) $\frac{\sin x \sin y + y}{\cos x \cos y - x}$ c) $\frac{y \sin(xy)}{x \sin(xy) - \cos y}$

3. Find $\partial z/\partial x$ and $\partial z/\partial y$.

a) $x^2 + y^2 + z^2 + 2x - 6z = 6$ b) $x^2 + \ln y = z^3$ c) $xyz + x + yz^2 = 0$

**A.** a) $\frac{\partial z}{\partial x} = -\frac{2x+2}{2z-6}$, $\frac{\partial z}{\partial y} = -\frac{2y}{2z-6}$ b) $\frac{\partial z}{\partial x} = -\frac{2x}{3z^2}$, $\frac{\partial z}{\partial y} = \frac{1}{3yz^2}$ c) $\frac{\partial z}{\partial x} = -\frac{yz+1}{xy+2yz}$, $\frac{\partial z}{\partial y} = -\frac{xz+z^2}{xy+2yz}$

4. If $u = f(x,y)$, where $x = e^s \cos t$ and $y = e^s \sin t$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2s}\left[\left(\frac{\partial u}{\partial s}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2\right].$$

Course Homework due Mar 5, Wed.

Feb 24, Mon. : **14.5** 1, 3, 5, 7, 9, 11, 13, 15, 21, 23

Feb 26, Wed. : **14.5** 25, 27, 29, 31, 33, 45, 58

Feb 28, Fri. : **14.6** 7, 9, 11, 13, 15, 39, 41, 43, 47, 49

5. If $z = f(x - y)$, show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.

6. Find the gradient of $f$, evaluate the gradient at the point $P$, and find the rate of change of $f$ at $P$ in the direction of the vector $\mathbf{u}$.

$$f(x, y, z) = \cos(xy) + z^2, \quad P = (2, 0, 1), \quad \mathbf{u} = \left\langle \frac{2}{7}, \frac{6}{7}, \frac{3}{7} \right\rangle$$

**A.** $\nabla f = (-y\sin(xy), -x\sin(xy), 2z)$, $\nabla f(P) = (0, 0, 2)$, $\nabla f(P) \cdot \mathbf{u} = \frac{6}{7}$

7. Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

$$f(x, y) = xe^y, \quad (2, 0), \quad \mathbf{v} = \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle.$$

**A.** $-5$

8. The **second directional derivative** of $f(x, y)$ is

$$D_{\mathbf{u}}^2 f(x, y) = D_{\mathbf{u}}[D_{\mathbf{u}} f(x, y)].$$

If $\mathbf{u} = \langle a, b \rangle$ is a unit vector and $f$ has continuous second partial derivatives, show that

$$D_{\mathbf{u}}^2 f = f_{xx}a^2 + 2f_{xy}ab + f_{yy}b^2.$$

Moreover, find the second directional derivative of $f(x, y) = e^x \cos y$ in the direction of $\mathbf{v} = \langle \frac{5}{13}, \frac{12}{13} \rangle$.

**A.** $\frac{25}{169}e^x \cos y - \frac{120}{169}e^x \sin y - \frac{144}{169}e^x \cos y$