1. Find an equation of the tangent plane to the given surface at the specified point.

$$z = \ln(x - 2y), \qquad (3, 1, 0)$$

A. $z = x - 2y - 1$

The Chain Rule

2. Use the Chain Rule to find dz/dt.

a) $z = x^3 + y^2 - x$, $x = e^t$, $y = \ln t$

A. $z_t = (3e^{2t} - 1)e^t + \frac{2\ln t}{t}$

b) $z = \sin(5x + y)$, $x = t^2$, $y = t$

A. $z_t = 10t\cos(5t^2 + t) + \cos(5t^2 + t)$

3. Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$.

a) $z = e^{x+3y}$, $x = s/t$, $y = t/s$

A. $z_s = e^{\frac{s}{t}+3\frac{t}{s}}(\frac{1}{t} - \frac{3t}{s^2})$, $\quad z_t = e^{\frac{s}{t}+3\frac{t}{s}}(\frac{-s}{t^2} + \frac{3}{s})$

b) $z = \sin\theta\cos\phi$, $\theta = s + t^2$, $\phi = s^2 + t$

A. $z_s = \cos(s + t^2)\cos(s^2 + t) - 2s\sin(s + t^2)\sin(s^2 + t)$

$z_t = 2t\cos(s + t^2)\cos(s^2 + t) - \sin(s + t^2)\sin(s^2 + t)$

c) $z = \tan(u/v)$, $u = 3s + 4t$, $v = s - 2t$

A. $z_s = \frac{3}{s-2t}\sec^2(\frac{3s+4t}{s-2t}) - \frac{3s+4t}{(s-2t)^2}\sec^2(\frac{3s+4t}{s-2t})$

$z_t = \frac{4}{s-2t}\sec^2(\frac{3s+4t}{s-2t}) + \frac{2(3s+4t)}{(s-2t)^2}\sec^2(\frac{3s+4t}{s-2t})$

Course Homework due Mar 5, Wed.

Feb 24, Mon. : **14.5** 1, 3, 5, 7, 9, 11, 13, 15, 21, 23

Feb 26, Wed. : **14.5** 25, 27, 29, 31, 33, 45, 58

Feb 28, Fri. : **14.6** 7, 9, 11, 13, 15, 39, 41, 43, 47, 49

4. Let $W(s,t) = F(u(s,t), v(s,t))$, where F, u, and v are differentiable and

$$\begin{array}{ll} u(1,0) = 2 & v(1,0) = 3 \\ u_s(1,0) = -2 & v_s(1,0) = 5 \\ u_t(1,0) = 6 & v_t(1,0) = 4 \\ F_u(2,3) = -1 & F_v(2,3) = 10 \end{array}$$

Find $W_s(1,0)$ and $W_t(1,0)$.

A. $W_s(1,0) = 52$, $W_t(1,0) = 34$

5. Use the Chain Rule to find the indicated partial derivatives.

$$\begin{array}{ll} w = xy + yz + z, & x = r\sin\theta, \quad y = r\cos\theta \quad z = \theta \\ \dfrac{\partial w}{\partial r}, \ \dfrac{\partial w}{\partial \theta} & \text{when } r = 1, \ \theta = \dfrac{\pi}{3} \end{array}$$

A. $w_r = \frac{\sqrt{3}}{2} + \frac{\pi}{6}$, $w_\theta = 1 - \frac{\pi}{2\sqrt{3}}$

6. If f is homogeneous of degree n, show that

$$f_x(tx, ty) = t^{n-1} f_x(x,y).$$

7. Find the gradient of f, evaluate the gradient at the point P, and find the rate of change of f at P in the direction of the vector $\mathbf{u}$.

$$f(x,y) = x^2y + xy, \quad P = (1,3), \quad \mathbf{u} = \frac{1}{2}(\mathbf{i} + \sqrt{3}\mathbf{j}).$$

A. $\nabla f = (2xy + y, x^2 + x)$, $\quad \nabla f(1,3) = (9,2)$, rate of change : $\frac{9}{2} + \sqrt{3}$.

Notice. **Quiz 5** will cover **limit** $\lim_{(x,y)\to(0,0)} f(x,y)$, **first partial derivatives** $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and finding an **equation of the tangent plane** at some specified point.