Solution 4

1. Find the unit tangent vector $\mathbf{T}(t)$ at the point with the given value of the parameter t.

a) $\mathbf{r}(t) = \langle 4t, t^2 - t, t^3 - \frac{3}{2}t^2 \rangle, \quad t = 2.$

Solution. First, a tangent vector is

$$\mathbf{r}'(t) = \langle 4, 2t-1, 3t^2 - 3t \rangle.$$

Thus, the unit tangent vector at $t = 2$ is

$$\mathbf{T}(1) = \frac{1}{\sqrt{4^2+3^2+6^2}} \langle 4, 3, 6 \rangle = \left\langle \frac{4}{\sqrt{61}}, \frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}} \right\rangle.$$

Answer. $\left\langle \frac{2}{\sqrt{61}}, \frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}} \right\rangle$.

b) $\mathbf{r}(t) = \ln t\mathbf{i} + 2\pi \sin(\frac{\pi}{2}t)\mathbf{j} + (t^4 + 4t)\mathbf{k}, \quad t = 1.$

Solution. Similarly as a),

$$\mathbf{r}'(t) = \frac{1}{t}\mathbf{i} + \pi^2 \cos(\frac{\pi}{2}t)\mathbf{j} + (4t^3 + 4)\mathbf{k}.$$

Thus, $\mathbf{r}'(1) = 1\mathbf{i} + 0\mathbf{j} + 8\mathbf{k}$ and

$$\mathbf{T}(1) = \frac{1}{\sqrt{1^2+0^2+8^2}} 1\mathbf{i} + 0\mathbf{j} + 8\mathbf{k}.$$

Answer. $\frac{1}{\sqrt{65}}\mathbf{i} + \frac{8}{\sqrt{65}}\mathbf{k}$

2. Find the length of the curve.

a) $\mathbf{r}(t) = \sin 3t\mathbf{i} + 4t\mathbf{j} + \cos 3t\mathbf{k}, \quad 1 \le t \le 4$

Solution. Note that $\mathbf{r}'(t) = 3\cos 3t\mathbf{i} + 4\mathbf{j} - 3\sin 3t\mathbf{k}$. Hence,

$$\begin{aligned} L &= \int_1^4 |r'(t)| dt \\ &= \int_1^4 \sqrt{9\cos^2 3t + 16 + 9\sin^2 3t} \\ &= \int_1^4 5 = 15. \end{aligned}$$

Answer. 15

b) $\mathbf{r}(t) = \langle \frac{8}{3}t^{\frac{3}{2}}, 4t, \frac{1}{2}t^2 - 3t \rangle, \quad 0 \le t \le 3$

Solution. We have $\mathbf{r}'(t) = \langle 4t^{\frac{1}{2}}, 4, t-3 \rangle$. Now, the length of the curve is

$$\begin{aligned} L &= \int_0^3 |r'(t)| dt \\ &= \int_0^3 \sqrt{16t + 16 + (t-3)^2} dt \\ &= \int_0^3 \sqrt{t^2 + 10t + 25} dt \\ &= \int_0^3 (t+5) dt = \frac{1}{2}(t+5)^2 \Big|_0^3 = \frac{39}{2} \end{aligned}$$

Answer. $\frac{39}{2}$

Letter grade for Quiz 4

$$\begin{aligned} 19.0 &< A0 \\ 18.0 &< A^- \le 19.0 \\ 17.0 &< B^+ \le 18.0 \\ 15.0 &< B0 \le 17.0 \\ 13.0 &< B^- \le 15 \\ 10.0 &< C^+ \le 13.0 \\ & C0 \le 10.0 \end{aligned}$$