# Solution 3

1. Determine whether the lines $L_1$ and $L_2$ are parallel, skew, or intersecting. If they intersect, find the point of intersection.

$$\text{a)} \begin{cases} L_1 : x = 1 + 6t, \quad y = 2 - 10t, \quad z = 3 + 4t \\ L_2 : x = 4 - 21s, \quad y = -5 + 35s, \quad z = 7 - 14s \end{cases} \qquad \text{b)} \begin{cases} L_1 : \dfrac{x-4}{5} = \dfrac{y-2}{7} = \dfrac{z-4}{-3} \\ L_2 : \dfrac{x-12}{-2} = \dfrac{y-11}{-5} = \dfrac{z-9}{11} \end{cases}$$

***Solution***. a) The slope of $L_1$ is $(6, -10, 4)$ and the slope of $L_2$ is $(-21, 35, -14)$. We know that $7 \cdot (6, -10, 4) = (42, -70, 28) = -2 \cdot (-21, 35, -14)$. Hence, $L_1$ and $L_2$ are parallel.

b) The slopes of $L_1$ and $L_2$ are $(5, 7, -3)$ and $(-2, -5, 11)$. Since those two vectors are not proportional to each other, $L_1$ and $L_2$ should be skew or intersecting. To check whether they are intersecting or not, we need to check the existence of $(t_0, s_0)$ such that

$$4 + 5t_0 = 12 - 2s_0, \qquad 2 + 7t_0 = 11 - 5s_0, \qquad 4 - 3t_0 = 9 + 11s_0.$$

Finding the solution, we have one when $t_0 = 2$ and $s_0 = -1$. In this case, the intersection point is $(x, y, z) = (14, 16, -2)$.

***Answer***. a) Parallel b) Intersecting at $(14, 16, -2)$

2. Find the cosine of the angle between the planes $x + 2y + 5z = 14$ and $3x - 2y - 7z = 1$.

***Solution***. The angle between the planes is same as the angle between the normal vectors of those planes. Thus, the cosine of the angle $\cos\theta$ is

$$\frac{(1,2,5) \cdot (3,-2,-7)}{|(1,2,5)| \cdot ||(3,-2,-7)|} = -\frac{18}{\sqrt{465}}.$$

***Answer***. $-\dfrac{18}{\sqrt{465}}$.

3. Find an equation of the plane.

   a) The plane through the point $(\frac{1}{3}, \frac{2}{5}, -3)$ and parallel to the plane $3x + 5y - 2z = 0$.

   ***Solution***. If two planes are parallel then their normal vectors are paralle. Thus, the normal vector of the plane we want to find is $(3, 5, -2)$. The plane also passes through $(\frac{1}{3}, \frac{2}{5}, -3)$. Hence, the equation for the plane is

$$(3, 5, -2) \cdot (x - \tfrac{1}{3}, y - \tfrac{2}{5}, z - (-3)) = 0.$$

   ***Answer***. $3x + 5y - 2z = 9$.

b) The plane consisting of all points that are equidistant from the points $(1,7,4)$ and $(-1,3,-2)$.

***Solution***. There are two ways to solve this problem. First, using the meaning of 'equidistant', we know that the equation for this plane is

$$\sqrt{(x-1)^2+(y-7)^2+(z-4)^2} = \sqrt{(x-(-1))^2+(y-3)^2+(z-(-2))^2}.$$

Squaring both sides, we get $4x+8y+12z=52$.

The other way to solve this is using a normal vector. First, the mid-point of $(1,7,4)$ and $(-1,3,-2)$ belongs to the plane. Note that the mid-point is $\frac{1}{2}((1,7,4)+(-1,3,-2)) = (0,5,1)$. Moreover, a vector from $(1,7,4)$ to $(-1,3,-2)$ is orthogonal to the plane, that is, $(-1,3,-2)-(1,7,4) = (-2,-4,-6)$ is a normal vector of the plane. Hence, the equation for the plane is

$$(-2,-4,-6)\cdot(x-0, y-5, z-1) = 0.$$

***Answer***. $x+2y+3z=13$.

c) The plane through the point $(5,-2,7)$ and contains the line of an equation $x-11 = \frac{y-2}{5} = 3z-1$.

***Solution***. Let the equation of the plane be $ax+by+cz=d$. Since the slope of the line is $(1,5,\frac{1}{3})$, we have

$$(a,b,c)\cdot(1,5,\tfrac{1}{3}) = 0. \qquad (1)$$

Next, since $(5,-2,7)$ and $(11,2,\frac{1}{3})$ are both on the plane, thus

$$5a-2b+7c = 11a+2b+\tfrac{1}{3}c. \qquad (2)$$

$(1)\times 20-(2)$ gives $26a+104b=0$, so $a:b=4:-1$. We may put $a=4$, $b=-1$. Then, from (1), we get $c=3$. Consequently, we can get $d$ by substituting $(5,-2,7)$ for $(x,y,z)$.

***Answer***. $4x-y+3z=43$.

4. Find equations for the surfaces obtained by rotating $x=y^2$ about the $x$-axis and $y$-axis, respectively.

***Solution***. Rotating about the $x$-axis, we need to substitute $\sqrt{y^2+z^2}$ for $y$. Since $x=y^2$ has no negative part for $x$, we do not need to square both sides.

$$x = y^2+z^2.$$

Around $y$-axis, change $x$ to $\sqrt{x^2+z^2}$.

$$y^2 = \sqrt{x^2+z^2}.$$

In fact, in this case, it has no difference if you square both sides. So, $y^4 = x^2+z^2$ would be also a right answer.

***Answer***. $x=y^2+z^2$ ($x$-axis case), $y^2=\sqrt{x^2+z^2}$ ($y$-axis case).

Letter grade for Quiz 3

$29.0 < A^+$
$24.0 < A0 \le 29.0$
$21.0 < A^- \le 24.0$
$18.0 < B^+ \le 21.0$
$16.0 < B0 \le 18.0$
$9.0 < B^- \le 16.0$
$< C^+ \le 9.0$