Quiz 10 (10mins, 20pts)

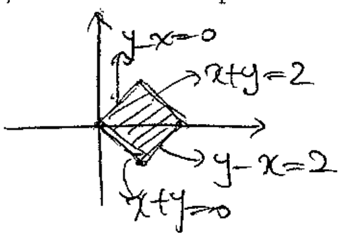
Please write down your name, SID, and solutions discernably.

Name : Dong Gyu Lim SID : Score :

1. (10pts) Evaluate the inegral by making an appropriate change of variables.

$$\iint_R xy\,dA$$

, where R is the square with vertices $(0,0)$, $(1,1)$, $(2,0)$, and $(1,-1)$.



Hence, $R=\{(x,y) : 0\le y-x\le 2,\ 0\le x+y\le 2\}$.

Let's think about change of variables.

$(x,y)\to(s,t)$ where $\begin{matrix} s=y-x \\ t=x+y \end{matrix} \Rightarrow x=\frac{t-s}{2}$, $y=\frac{t+s}{2}$

Then, the Jacobian of this transformation is

$$\begin{vmatrix} \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} \end{vmatrix} = \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = \left|-\frac{1}{2}\right| = \frac{1}{2}.$$

$$\therefore \iint_R xy\,dA = \iint_R \frac{1}{2}\,\frac{t-s}{2}\cdot\frac{t+s}{2}\,dt\,ds = \int_0^2\int_0^2 \frac{1}{8}(t^2-s^2)\,dt\,ds.$$

$$= \int_0^2 \frac{1}{8}\cdot\left(\frac{1}{3}t^3-ts^2\right)\Big|_0^2$$

$$= \int_0^2 \frac{1}{3}-\frac{1}{4}s^2 = \frac{2}{3}-\frac{1}{12}s^3\Big|_0^2$$

$$= \frac{2}{3}-\frac{2}{3}=0.$$

Answer. 0.

2. (10pts) Evaluate the line integral

$$\int_C (x^2 + y^2 + z^2)ds$$

, where $C : x = 3t,\ y = \cos 4t,\ z = \sin 4t,\ 0 \le t \le 2\pi$.

$$\int_C (x^2+y^2+z^2)ds = \int_0^{2\pi} ((3t)^2 + \cos^2 4t + \sin^2 4t) \cdot |(3, -4\sin 4t, 4\cos 4t)|\, dt$$

$$|(3, -4\sin 4t, 4\cos 4t)| = \sqrt{9 + 16\sin^2 4t + 16\cos^2 4t} = \sqrt{25} = 5.$$

$$= \int_0^{2\pi} (9t^2 + 1) \cdot 5 = (15t^3 + 5t)\Big|_0^{2\pi} = 120\pi^3 + 10\pi.$$

Answer. $120\pi^3 + 10\pi$.