Quiz 10 (10mins, 20pts)

Please write down your name, SID, and solutions discernably.

Name : DongGyu Lim SID : Score :

1. **(10pts)** Evaluate the inegral by making an appropriate change of variables.

$$\iint_R (x+y)e^{x^2-y^2}\,dA$$

, where R is the rectangle enclosed by the lines $x-y=0$, $x-y=2$, $x+y=0$, and $x+y=3$.

Let's think about the transformation.

$(x,y) \longrightarrow (t,s)$

where $t = x-y$, $s = x+y$. Then, $x = \frac{t+s}{2}$, $y = \frac{s-t}{2}$.

Hence, the Jacobian of transformation is

$$\begin{vmatrix} \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2}.$$

$$\therefore \iint_R (x+y)e^{x^2-y^2}dA = \iint_R \frac{1}{2}s\cdot e^{ts}\,dt\,ds = \int_0^3\int_0^2 \frac{1}{2}se^{ts}\,dt\,ds$$

$$= \int_0^3 \frac{1}{2}e^{ts}\Big|_0^2 ds = \int_0^3 \frac{1}{2}(e^{2s}-1)\,ds$$

$$= \left(\frac{1}{4}e^{2s} - \frac{s}{2}\right)\Big|_0^3$$

$$= \frac{1}{4}e^6 - \frac{3}{2} - \frac{1}{4}$$

$$= \frac{1}{4}(e^6 - 7).$$

Answer: $\frac{1}{4}(e^6-7)$.

2. (10pts) Evaluate the line integral

$$\int_C (x^2+y^2+z^2)ds$$

, where $C: x=t,\ y=\cos 2t,\ z=\sin 2t,\ 0 \le t \le 2\pi$.

$$\int_C (x^2+y^2+z^2)ds = \int_0^{2\pi} (t^2+\cos^2 2t+\sin^2 2t)\cdot \underbrace{|(1,-2\sin 2t, 2\cos 2t)|}_{\sqrt{1+4\sin^2 2t+4\cos^2 2t}=\sqrt{5}} dt$$

$$= \sqrt{5}\int_0^{2\pi} (t^2+1) = \sqrt{5}\cdot\left(\frac{1}{3}t^3+t\right)\Big|_0^{2\pi}$$

$$= \frac{8\sqrt{5}}{3}\pi^3+2\sqrt{5}\pi.$$

Answer. $\frac{8\sqrt{5}}{3}\pi^3+2\sqrt{5}\pi.$