QUIZ 9 (35MINS, 40PTS)

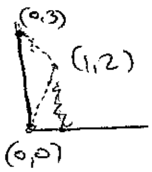
Please write down your name, SID, and solutions discernably.

Name : DongGyu Lim. SID : Score :

1. **(10pts)** Evaluate the double integral,

$$\iint_D (2xy)\,dA$$

where D is the triangular region with vertices $(0,0)$, $(1,2)$, and $(0,3)$.



We shall use vertical segments, that is, first fix x. : $0 \le x \le 1$.
Note that two dotted lines have equations $y=2x$ and $x+y=3$.
Hence, for each x, $2x \le y \le 3-x$.

$$\therefore \iint_D (2xy)\,dA = \int_0^1 \int_{2x}^{3-x} 2xy\,dy\,dx = \int_0^1 (x \cdot y^2)\Big|_{2x}^{3-x} dx$$

$$= \int_0^1 x\left((3-x)^2-(2x)^2\right)dx$$

$$= \int_0^1 (-3x^3-6x^2+9x)\,dx$$

$$= \left(-\frac{3}{4}x^4 - 2x^3 + \frac{9}{2}x^2\right)\Big|_0^1$$

$$= -\frac{3}{4} - 2 + \frac{9}{2} - 0 = \frac{18-8-3}{4} = \frac{7}{4}$$

Answer.

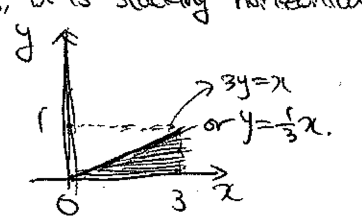
2. **(10pts)** Sketch the region of integration and change the order of integration.

$$\int_0^1 \int_{3y}^3 f(x,y)\,dx\,dy$$

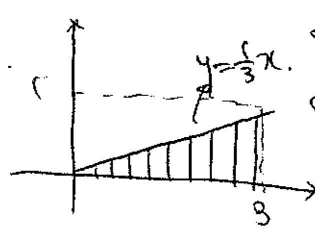
The region has its representation.

$$D = \{(x,y) : 0 \le y \le 1,\ 3y \le x \le 3\}.$$

So, it is stacking horizontal segments.



Changing the order of integration



stacking vertical segments.
So, first fix x.
$0 \le x \le 3$.
and then
$0 \le y \le \frac{1}{3}x$

$$\therefore \int_0^3 \int_0^{\frac{1}{3}x} f(x,y)\,dy\,dx$$ Answer.

3. (10pts) Evaluate the given integral by changing to polar coordinates,

$$\iint_R \frac{y^2}{x^2+y^2}\,dA$$

where R is the region that lies between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ with $0<a<b$.

The region R in polar coordinates is given as $a^2 < \underbrace{x^2+y^2}_{= r^2} < b^2 \Rightarrow a<r<b$.

$$\therefore \iint_R \frac{y^2}{x^2+y^2}\,dA = \int_a^b \int_0^{2\pi} \frac{r^2\sin^2\theta}{r^2}\cdot r\,dr\,d\theta \quad \text{since } dx\,dy = r\,dr\,d\theta.$$

$$= \int_a^b r\,dr \cdot \int_0^{2\pi} \sin^2\theta\,d\theta$$

$$= \frac{1}{2}(b^2-a^2)\cdot \int_0^{2\pi} \frac{1-\cos 2\theta}{2}\,d\theta = \frac{1}{2}(b^2-a^2)\cdot\left(\frac{2\pi}{2} - \underbrace{\left(\frac{\sin 2\theta}{4}\right)\Big|_0^{2\pi}}_{=0}\right)$$

$$= \frac{\pi}{2}(b^2-a^2).$$

Answer

4. (10pts) Evaluate $\iiint_H (9-x^2-y^2)\,dV$, where H is the solid hemisphere $x^2+y^2+z^2\le 9$, $z\ge 0$.

You may use spherical coordinates, though I shall use cylindrical coordinates.

In this case, one good thing is that $x^2+y^2=r^2$ is simple and has no trigonometric ftns.

On the one hand, the region has ~~ex~~ an expression with square root.

$$\Rightarrow 0\le z\le \sqrt{9-x^2-y^2} = 0\le z\le\sqrt{9-r^2}$$

Anyway, $\iiint_H (9-x^2-y^2)\,dV = \int_0^{2\pi}\int_0^3\int_0^{\sqrt{9-r^2}} (9-r^2)\cdot r\,dz\,dr\,d\theta$

$$= 2\pi\cdot\int_0^3 (9-r^2)^{3/2}\cdot r\,dr.$$

Note that $(9-r^2)' = -2r$

$\therefore$ $s = 9-r^2 \Rightarrow$

$$2\pi\int_9^0 s^{3/2}\cdot -\frac{1}{2}ds = -\pi\int_9^0 s^{3/2}\,ds$$

$$= -\frac{2}{5}\pi\, s^{5/2}\Big|_9^0$$

$$= \frac{2\pi}{5}\cdot 9^{5/2} = \frac{2\cdot 3^5}{5}\pi.$$

Answer