1. Show that any vector field of the form

$$\mathbf{F}(x, y, z) = f(x)\mathbf{i} + g(y)\mathbf{j} + h(z)\mathbf{k}$$

where f, g, h are differentiable functions, is irrotational.

2. Prove the identity, assuming that the appropriate partial derivatives exist and are continuous. If f is a scalar field and $\mathbf{F}$, $\mathbf{G}$ are vector fields, then $f\mathbf{F}$, $\mathbf{F} \cdot \mathbf{G}$, and $\mathbf{F} \times \mathbf{G}$ are defined by

$$(f\mathbf{F})(x, y, z) = f(x, y, z)\mathbf{F}(x, y, z)$$
$$(\mathbf{F} \cdot \mathbf{G})(x, y, z) = \mathbf{F}(x, y, z) \cdot \mathbf{G}(x, y, z)$$
$$(\mathbf{F} \times \mathbf{G})(x, y, z) = \mathbf{F}(x, y, z) \times \mathbf{G}(x, y, z)$$

a) $\mathrm{div}(f\mathbf{F}) = f\mathrm{div}\mathbf{F} + \mathbf{F} \cdot \nabla f$

b) $\mathrm{curl}(f\mathbf{F}) = f\mathrm{curl}\mathbf{F} + (\nabla f) \times \mathbf{F}$

c) $\mathrm{div}(\mathbf{F} \times \mathbf{G}) = \mathbf{G}\cdot\mathrm{curl}\mathbf{F} - \mathbf{F}\cdot\mathrm{curl}\mathbf{G}$

d) $\mathrm{div}(\nabla f \times \nabla g) = 0$

3. Find a parametric representation for the surface.

   a) The part of the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ that lies to the left of the xz-plane.

   b) The part of the sphere $x^2 + y^2 + z^2 = 16$ that lies between the planes $z = -2$ and $z = 2$.

4. Find an equation of the tangent plane to the given parametric surface at the specified point.

$$\mathbf{r}(u, v) = u^2\mathbf{i} + 2u \sin v\mathbf{j} + u \cos v\mathbf{k}, \quad u = 1,\ v = 0$$

5. Find the area of the part of the plane with vector equation

$$\mathbf{r}(u, v) = \langle u + v, 2 - 3u, 1 + u - v\rangle$$

   that is given by $0 \leq u \leq 2$, $-1 \leq v \leq 1$.

Course Homework due Apr 30, Wed.

Apr 21, Mon. : **16.6** 13-18, 19, 21, 37, 41, 43
Apr 23, Wed. : **Midterm #2**
Apr 25, Fri. : **16.7** 5, 7, 9, 11, 13, 15, 17