QUIZ 11 (20MINS, 30PTS)

Please write down your name, SID, and solutions discernably.

Name : DongGyu Lim SID : Score :

1. **(10pts)** Evaluate the line integral.

$$\int_C xye^{yz}\,dy,$$

where $C: x = t,\ y = t^2,\ z = t^3,\ 0 \le t \le 1.$

We are given a parametrization of C.

By def'n of $\int dy$,

$$\int_C xye^{yz}dy = \int_0^1 t\cdot t^2\cdot e^{t^2\cdot t^3}\cdot 2t\,dt = \int_0^1 2t^4\cdot e^{t^5}dt$$

$$\overset{u=t^5,\ du=5t^4dt}{=} \int_0^1 \frac{2}{5}e^u du = \frac{2}{5}(e-1)$$

Answer. $\frac{2}{5}(e-1)$

2. **(10pts)** Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.

$$\mathbf{F}(x, y, z) = (x-y)\mathbf{i} + y^2\mathbf{j} + (z-x)\mathbf{k},$$

where C is given by the vector function $\mathbf{r}(t) = t^3\mathbf{i} - t^2\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$.

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \left(t^3-(-t^2),\ (-t^2)^2,\ t-t^3\right)\cdot\left(3t^2, -2t, 1\right)dt$$

$$= \int_0^1 (3t^5+3t^4-2t^5+t-t^3)dt = \int_0^1 (t^5+3t^4-t^3+t)\,dt$$

$$= \left(\frac{1}{6}t^6+\frac{3}{5}t^5-\frac{1}{4}t^4+\frac{1}{2}t^2\right)\Big|_0^1$$

$$= \frac{1}{6}+\frac{3}{5}-\frac{1}{4}+\frac{1}{2} = \frac{61}{60}$$

Answer. $\frac{61}{60}$

3. (10pts) Find a function f such that $\mathbf{F} = \nabla f$ and use f to evaluate $\int_C \mathbf{F} \cdot dr$ along the given curve C.

$$\mathbf{F}(x, y) = (1 + xy)e^{xy}\mathbf{i} + x^2 e^{xy}\mathbf{j}, \quad C : \mathbf{r}(t) = \cos t\mathbf{i} + 2\sin t\mathbf{j}, \quad 0 \le t \le \frac{\pi}{2}$$

From the fact that $\frac{\partial f}{\partial y} = x^2 e^{xy}$, we might guess $f(x,y) = xe^{xy} +$ function of x.

But, it turns out that $f(x,y) = xe^{xy}$ satisfies $\vec{F}(x,y) = \nabla f(x,y)$.

Obviously, f is defined on the whole plane $\mathbb{R}^2$ which is simply connected.

Hence we can apply Fundamental Theorem of Line Integrals so that

$$\begin{aligned}\int_C \vec{F} \cdot d\vec{r} &= f(r(\tfrac{\pi}{2})) - f(0) \\ &= f((0,2)) - f(1,0) \\ &= 0 - 1 \cdot e^0 = -1.\end{aligned}$$

Answer. -1.