QUIZ 11 (20MINS, 30PTS)

Please write down your name, SID, and solutions discernably.

Name : Dong Gyu Lim SID : Score :

1. (10pts) Evaluate the line integral.

$$\int_C e^x dx,$$

where C is the arc of the curve $x = y^3$ from $(-1,-1)$ to $(1,1)$.

Let's parametrize "$x=y^3$ from $(-1,-1)$ to $(1,1)$" as $r(t)=(t^3,t)$, $-1\le t\le 1$.

By def'n of line integrals

$$\int_C e^x dx = \int_{-1}^{1} e^{t^3}\cdot 3t^2 dt \underset{\substack{u=t^3\\ \Rightarrow du=3t^2dt}}{=} \int_{-1}^{1} e^u du = e^u\Big|_{-1}^{1} = e-e^{-1}.$$

Answer. $e-\frac{1}{e}$

2. (10pts) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.

$$\mathbf{F}(x,y) = (x+y)\mathbf{i} + (y-z)\mathbf{j} + z^2\mathbf{k},$$

where C is given by the vector function $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + t^2\mathbf{k}$, $0 \le t \le 1$.

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \vec{F}(r(t))\cdot r'(t)\,dt$$

$$= \int_0^1 (t^2+t^3, t^3-t^2, t^4)\cdot(2t, 3t^2, 2t)\,dt$$

$$= \int_0^1 (2t^3+2t^4+3t^5-3t^4+2t^5)\,dt = \int_0^1 (5t^5-t^4+2t^3)\,dt$$

$$= \left(\frac{5}{6}t^6-\frac{1}{5}t^5+\frac{1}{2}t^4\right)\Big|_0^1 = \frac{5}{6}-\frac{1}{5}+\frac{1}{2} = \frac{17}{15}$$

Answer. $\frac{17}{15}$

3. (10pts) Find a function f such that $\mathbf{F} = \nabla f$ and use f to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.

$$\mathbf{F}(x,y,z) = yze^{xz}\mathbf{i} + e^{xz}\mathbf{j} + xye^{xz}\mathbf{k}, \quad C : \mathbf{r}(t) = (t^2+1)\mathbf{i} + (t^2-1)\mathbf{j} + (t^2-2t)\mathbf{k}, \quad 0 \le t \le 2$$

Let $f(x,y,z) = ye^{xz}$, then $\vec{F} = \nabla f$ (can easily check)

Since f is defined on the whole plane $\mathbb{R}^2$ which is simply connected, by Fundamental Theorem of Line Integrals,

$$\int_C \vec{F} \cdot d\vec{r} = f(r(2)) - f(r(0))$$
$$= f((5,3,0)) - f((1,-1,0))$$
$$= 3 \cdot e^0 - (-1) \cdot e^0 = 4.$$

Answer. 4.