6. radially symmetric sol'ns

$$\Delta u = k^2 u$$

$$\| $$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u}{\partial r}\right)$$ (in case u is radially sym.)

$$\|$$

$$u_{rr} + \frac{2}{r} u_r$$

$$\|$$

$$\frac{1}{r}(ru)_{rr}$$

$$\frac{1}{r}\cdot\frac{\partial^2 (ru)}{\partial r^2} = k^2 u$$

defining $v = ru$

$$v_{rr} = k^2 v$$

$$v(r) = c_1 e^{kr} + c_2 e^{-kr}$$

$$\Rightarrow u(r) = c_1 \cdot \frac{e^{kr}}{r} + c_2 \cdot \frac{e^{-kr}}{r}.$$

7. LHS: $u_{rr} + \frac{1}{r} u_r$ $\quad k^2 u =$ RHS

$$\ne$$

$$\cancel{\frac{1}{r}(\ldots)}$$

$r^2 u'' + r u' + (r^2 - k^2) u = 0$

$$r^2 u'' + r u' \boxed{-} k^2 r^2 u = 0.$$

$r^2 u''(r) + r u'(r) - k^2 r^2 u(r) = 0$

Bessel's eqn of order 0

$r \leftarrow cr$

$v(r) := u(cr)$ for some const c.

$$r^2 v'' + r v' \underbrace{- k^2 c^2}_{1} r^2 v = 0.$$

$c = \frac{1}{k} i$ or $\frac{1}{ki}$.

$\Rightarrow$ Bessel's eqn. of order 0.

8. maximum principle can be applied?

$\boxed{U : \text{harmonic on}}$ U (open set) $\overset{\shortparallel}{\text{interior of } D}$

+ u is continuous on ∂D.

$\Longrightarrow$ $\boxed{\text{max. principle}}$ is valid.

1st method

1) $\boxed{U_{xx} + U_{yy} = 0}$

2nd method

Guess $f(z)$ hol. s.t. $\text{Re} f = u$. $\quad \frac{f + \bar{f}}{2}$

$$\frac{1 - x^2 - y^2}{(1-x)^2 + y^2} = \frac{\boxed{1 - z\bar{z}}}{|z-1|^2 = (z-1)\overline{(z-1)}}$$

$f(z) = \frac{1}{z-1}$ might work. $\Rightarrow f = \frac{a}{z-1} + b$ for some a, b.

$f + \bar{f}$ $\quad$ $\text{Re} f = u$.

Is hol. in $\{|z| < 1\}$.

- on ∂D?

$\downarrow$

$x^2 + y^2 = 1$

$$u(x,y) = \frac{1-1}{1 - 2x + 1} = 0$$

$\boxed{x = 1}$.

In fact, $u(x,y)$ is not def'd at $x = 1$. $\Rightarrow$ not conti.

9. $u_{xx}+u_{yy}=0$ on $D=\{ |x|\le 1 \}$

$u=1+3\sin\theta$ on ∂D.

Sep. of Variables,

$$\Rightarrow u(r,\theta)=\frac{A_0}{2}+\sum_{n=1}^{\infty} r^n \left(A_n\cos n\theta + B_n\sin n\theta\right)$$

$$A_n, B_n = \frac{1}{\pi}\int_0^{2\pi} \text{boundary fn.} \cdot \begin{matrix}\cos n\theta \\ \sin n\theta\end{matrix}\, d\theta$$

$\Rightarrow$ Compute A_n, B_n

$\Rightarrow \boxed{u=1+3y}$ $\quad u_{xx}+u_{yy}=0$

Double check if u is harmonic.