Welcome to Math 110
Section 112

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math

10. Final Grade Components

NOT just answers but reasonings.

10.1. Grading scheme. Grades will be computed by taking:

(1) 5% homework. There will be ≈ 37 HWs, each out of 10 points, checked for completeness.

gradescope

(2) 14% quizzes (using only the top 10 quiz scores). At the instructor's discretion, quiz medians of all sections in the class will be uniformized at the end.

gradescope

(3) 12% lecture attendance. There will be 2 polls in lecture or alternative late evening quiz for "overseas" and DSP students with lecture accommodations. Each poll is out of 2 pts:
- answer the two polls any way during the given time: 1 pt;
- answer at least one poll correctly: 1 pt.

The top 30 (out of ≈ 38) lecture attendances will count towards the final grade.

(4) 4% discussion attendance. There will be 2 polls in discussion sections (no alternative options for section polls). Each poll is out of 2 pts:
- answer the two polls any way during the given time: 1 pt;
- answer at least one poll correctly: 1 pt.

The top 10 (out of ≈ 14) section attendances will count towards the final grade.

just be here and focus.

(5) 20% each midterm 1.

(6) 20% each midterm 2.

(7) 25% final exam.

POLL

2. Let V be the set of matrices of the form $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$, $a, b \in \mathbb{R}$. Prove or disprove that V is a vector space (with addition and scalar multiplication of matrices defined in the standard way).

Hint. Finding one counterexample would be enough.

- $3 \cdot \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} = \begin{pmatrix} 3 & 3a \\ 3b & 3 \end{pmatrix}$ is not of the form $\begin{pmatrix} 1 & * \\ * & 1 \end{pmatrix}$.

- No zero vector.

  $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \notin V$.

- $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} + \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} = \begin{pmatrix} 2 & a+c \\ b+d & 2 \end{pmatrix}$

2. Let F be a field.

(i) Prove that $W_1 = \{(a_1, a_2, \ldots, a_n) \in \mathsf{F}^n \mid a_1 + a_2 + \cdots + a_n = 0\}$ is a subspace of F^n.

(ii) Prove that $W_2 = \{(a_1, a_2, \ldots, a_n) \in \mathsf{F}^n \mid a_1 + a_2 + \cdots + a_n = 1\}$ is not a subspace of F^n.

What are the conditions for a subset to be a subspace?

- zero vector
- closed under add.
  scalar mult.
-

$\boxed{\text{VS } 1 \sim 8}$.

- commutativity
- distributivity . . . .

$W_1 \subseteq F^n \leftarrow$ vec. sp.

$(0, 0, 0, \ldots, 0)$ is the zero vector of F^n.

But it is not in W_2 b/c

$0 + \cdots + 0 \neq 1$.

2. **(Don't try this at home alone!)** Let F be a field, and let $\mathsf{P}(\mathsf{F})$ be the vector space of all polynomials with coefficients in F. There is a natural function γ, defined by

$$\begin{aligned} \gamma : \mathsf{P}(\mathsf{F}) &\to \mathcal{F}(\mathsf{F}, \mathsf{F}) \\ p(x) &\mapsto (\beta \mapsto p(\beta)). \end{aligned}$$

Explicitly, if $p(x) = a_n x^n + \cdots + a_1 x + a_0 \in \mathsf{P}(\mathsf{F})$, then $\gamma(p) : \mathsf{F} \to \mathsf{F}$ is the function defined by the formula

$$\gamma(p)(\beta) = a_n \beta^n + \cdots + a_1 \beta + a_0$$

for any $\beta \in \mathsf{F}$. Simply put, $\gamma(p)$ is the polynomial p, now considered as a function by evaluating p at elements of F. If $\mathsf{F} = \mathbb{Q}, \mathbb{R}$, or $\mathbb{C}$, then the function γ is one-to-one[2] (see Appendix B). Is γ always one-to-one?

Simply a vector versus A function

One-to-one?

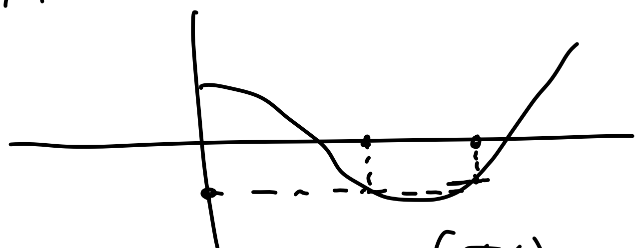
$\gamma : X \to Y$

If $\gamma(x_1) = \gamma(x_2)$ implies $x_1 = x_2$.

then γ is one-to-one.

POLL

$f(x) = \cos x$



$\cos\left(\pi/2\right) = 0 = \cos\left(3\pi/2\right)$

$f(x) = x$: 1-1.

$f(x) = x^2$: NOT 1-1.

$f(-1) = f(1)$

$-1 \neq 1.$

1. **(True/False Jeopardy)** Supply convincing reasoning for your answer.

$\exists \bar{0}$.

(a) T F A vector space is nonempty.

(b) T F Any vector space over $\mathbb{C}$ is a vector space over $\mathbb{R}$.

(c) T F The intersection of two subspaces of a vector space is a vector space.

(d) T F The union of two subspaces of a vector space is a vector space.

(e) T F Any vector space has at least two distinct subspaces.

VS1~8 VS7

- zw
- sca.
- S

$\mathbb{C}/\mathbb{C}$ $\{1\}$

$\mathbb{C}/\mathbb{R}$ $\{1, i\}$.